Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Find the radius of convergence and interval of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}.$$

Find the radius of convergence and interval of convergence for

$$\sum_{n=0}^{\infty} n!(x-3)^n .$$

Derivatives and Integrals for Power Series

Expand $\sum_{n=0}^{\infty} a_n x^n$

Now, what happens when we take the derivative of this?

Thm – If $\sum_{n=0}^{\infty} a_n x^n$ converges on (-c, c) then $\sum_{n=0}^{\infty} \frac{d}{dx}\left(a_n x^n\right)$ converges on (-c, c) (you still must check the endpoints for each problem)

Example:

Find the derivative of $\sum_{n=0}^{\infty} \frac{3nx^n}{n^2+1}$

Integration of Series:

Thm – If $f(x)=\sum_{n=0}^{\infty} a_n x^n$ converges on (-c, c), then $g(x)=\sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$ converges on (-c, c) and $\int f(x)dx = g(x)+C$

Find a power series for $\tan^{-1} x$ using integration.

Integrate $\int \sum_{n=0}^{\infty} \frac{3nx^n}{n^2+1}\,dx$

(9.8) Definition of nth degree Taylor polynomial centered at c:

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the nth degree Taylor polynomial for f at c.

Give the 8^{th} degree Taylor polynomial approximation to $y = e^x$ centered at $x = 0$.

k	$f^k(x)$	$f^k(1)$	$\dfrac{f^k(1)}{k!}$	term

Find an n[th] degree Taylor polynomial approximation for $f(x) = \cos(x)$ centered at x = 0.

k	$f^k(x)$	$f^k(0)$	$\frac{f^k(0)}{k!}$	term

Find an n^{th} degree Taylor polynomial approximation for $f(x) = \sin(x)$ centered at x = 0.

k	$f^k(x)$	$f^k(0)$	$\dfrac{f^k(0)}{k!}$	term

Use the fourth-degree Taylor approximation $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ for x near 0 to find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

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1. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = e^x$ centered at $x = 0$.

2. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = \sin(x)$ centered at $x = 0$.

3. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = \cos(x)$ centered at $x = 0$.

4. Give the coefficient of x^{10} for the 11^{th} degree Taylor polynomial approximation to $\sin(x)$ centered at $x = 0$.

5. Give the coefficient of $(x + 1)^2$ for the 4^{th} degree Taylor polynomial approximation to x^4 centered at $x = -1$.

k	$f^k(x)$	$f^k(-1)$	$\frac{f^k(-1)}{k!}$	term

6. Give the 3rd degree Taylor polynomial for $f(\mathrm{x}) = x^3 - 1$ centered at $x = 1$.