Math 1432

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Office Hours:

Mondays 1-2pm,
Fridays noon-1pm
(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

Popper 28

1. The series $\sum \frac{(-1)^k \sqrt{k+2}}{\sqrt{4k^3+2k+1}}$ is

Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n \cdot 2^n}$

Popper

2. Find the radius of convergence for the power series $\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$

3. Find the interval of convergence for the power series: $\sum_{n=0}^{\infty}\frac{1}{3^n}(x-1)^n$

Find $\frac{d}{dx}\sum\frac{(-1)^k x^k}{3k^2+1}$

4. Find $\dfrac{d}{dx}\sum \dfrac{(-1)^k 2^k}{k^2+1}x^k$

Find $\int \sum \frac{(-1)^k 3^k x^k}{k^2}\, dx$

5. Find $\int \sum \frac{(-1)^k 2^k}{k^2+1} x^k \, dx$

Taylor Polynomials in x
Taylor Series in x

There are many functions that we only know at one point, or a handful of isolated points. Such as the trigonometric functions, e^x, *ln* x, etc.

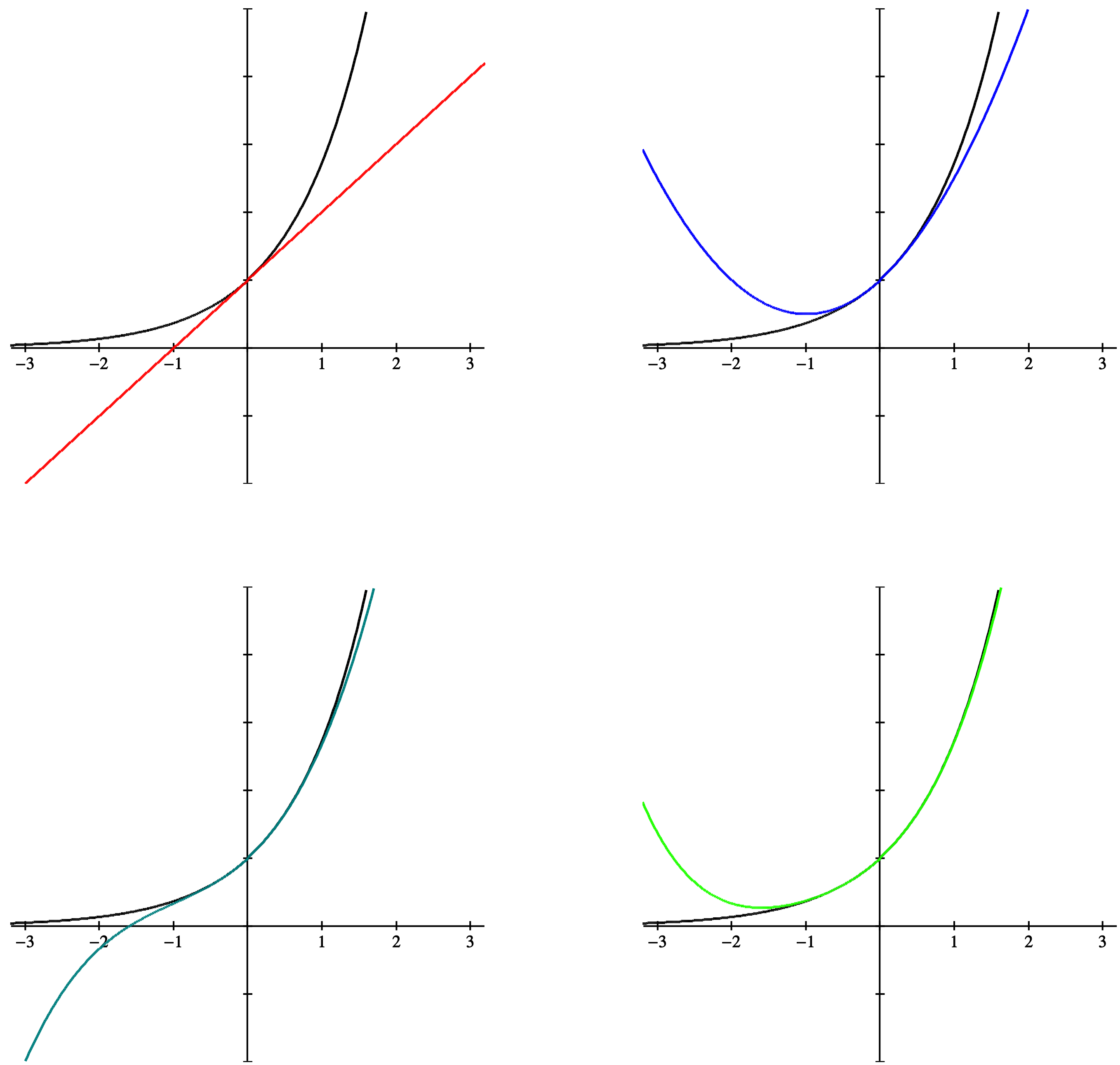
Let's create a polynomial P(x) that has the same properties as some function *f* (x) that we know very well at x = a, such as *sin* (x) or e^x around x = 0.

The properties that we need to consider are the function and derivative properties.

Why a polynomial?

1) Find a polynomial of degree n = 4 for $f(x) = e^{2x}$ about x = 0.

k	$f^k(x)$	$f^k(0)$	$\dfrac{f^k(0)}{k!}$	term


−3 −2 −1 1 2 3
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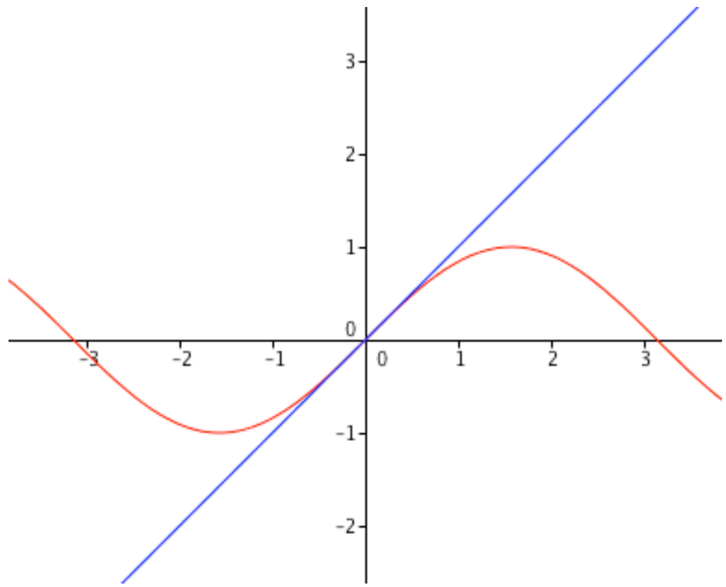
−3 −2 −1 1 2 3

2) Find a polynomial of degree n = 5 for $f(x) = \sin x$ about x = 0.

k	$f^k(x)$	$f^k(0)$	$\frac{f^k(0)}{k!}$	term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-2
-1
0
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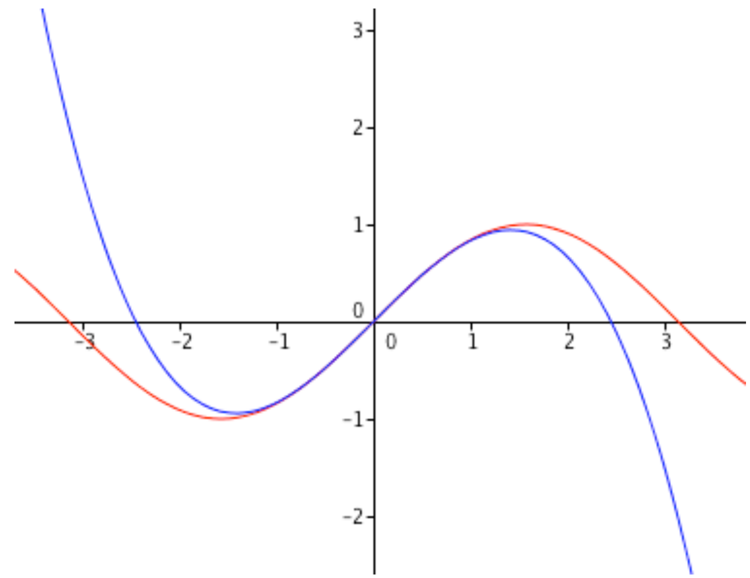
3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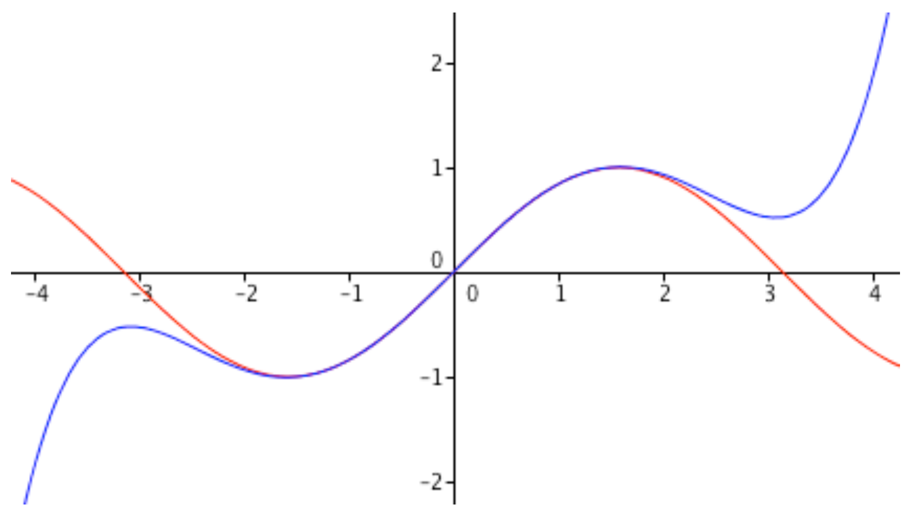
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1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4

3) Find a polynomial of degree n = 4 for $f(x) = \ln|x+1|$ about x = 0.

k	$f^k(x)$	$f^k(0)$	$\frac{f^k(0)}{k!}$	term

4) Use the Taylor approximation $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ for x near 0 to find:

$$\lim_{x \to 0} \frac{e^x - 1}{2x}.$$

5) Use the Taylor approximation $\sin x \approx x - \frac{x^3}{3!}$ for x near 0 to find

$$\lim_{x \to 0} \frac{\sin x}{x}.$$

Definition of nth degree Taylor polynomial:

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the nth degree Taylor polynomial for f at c.

If c = 0, then

$$P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \ldots + \frac{f^{(n)}(0)}{n!}(x)^n$$

may be called the nth degree **Maclaurin** polynomial for f.

6) Give the 8^{th} degree Taylor polynomial approximation to ln (x) centered at $x = 1$.

k	$f^k(x)$	$f^k(1)$	$\frac{f^k(1)}{k!}$	term