Math 1432

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Office Hours:

Mondays 1-2pm,
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(also available by appointment)

Class webpage:

http://www.math.uh.edu/~bekki/Math1432.html

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1. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = e^x$ centered at $x = 0$.

2. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = \sin(x)$ centered at $x = 0$.

3. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = \cos(x)$ centered at $x = 0$.

4. Give the 7^{th} degree Taylor polynomial approximation for $f(x) = \ln(x+1)$ centered at $x = 0$.

5. Give the coefficient of $(x - 1)^3$ for the 8^{th} degree Taylor polynomial approximation to $\ln(x)$ centered at $x = 1$.

Find the Taylor polynomial of degree n = 5 for $f(x) = ln\,x$ at c = 1.
Then use $P_5(x)$ to approximate the value of $ln(1.1)$.

k	$f^k(x)$	$f^k(0)$	$\frac{f^k(0)}{k!}$	term
	$f(x) = ln\,x$			
	$f'(x) = x^{-1}$			
	$f''(x) = -1x^{-2}$			
	$f'''(x) = 2x^{-3}$			
	$f^{(4)}(x) = -6x^{-4}$			
	$f^{(5)}(x) = 24x^{-5}$			

Suppose that g is a function which has continuous derivatives, and that

$$g(2)=3, \quad g'(2)=-4, \quad g''(2)=7, \quad g'''(2)=-5.$$

Write the Taylor polynomial of degree 3 for g centered at x = 2.

Find $P_6(x)$ for $f(x) = x^2 \cos(5x)$

Find $f^{(15)}(0)$ for $f(x) = e^{x^3}$

Lagrange Form of the Remainder
or
Lagrange Error Bound or Taylor's Theorem Remainder

When a Taylor polynomial is used to approximate a function, we need a way to see how accurately the polynomial approximates the function.

$f(x) = P_n(x) + R_n(x)$ so $R_n(x) = f(x) - P_n(x)$

Written in words:

Function = Polynomial + Remainder

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

so

Remainder = Function – Polynomial

Lagrange Formula for Remainder:

Suppose f has n+1 continuous derivatives on an open interval that contains 0. Let x be in that interval and let $P_n(x)$ be the n^{th} Taylor Polynomial for f. Then

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$$

where c is some number between 0 and x.

If we rewrite Taylor's theorem using the Lagrange formula for the remainder, we have

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$$

where c is some number between 0 and x.

If there is a number M so that $\left|f^{(n+1)}(c)\right| \le M$

for all c between 0 and x then $\left|f(x) - P_n(x)\right| \le \dfrac{M}{(n+1)!}|x|^{n+1}$

or

$\left|R_n(x)\right| \le \left(max\left|f^{(n+1)}(c)\right|\right)\dfrac{|x|^{n+1}}{(n+1)!}$ for c between 0 and x.

We probably will not know the value of c.

Give an error estimate for the approximation of *sin* (x) by $P_9(x)$ for an arbitrary value of x between 0 and π/4, centered at x = 0.

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = -\sin x$$

$$f'''(x) = -\cos x$$

$$f^{(4)}(x) = \sin x$$

Give an error estimate for the approximation of *cos* (x) by $P_{10}(x)$ for an arbitrary value of x between 0 and π/4, centered at x = 0.

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

$$f^{(4)}(x) = \cos x$$

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6. Assume that *f(x)* is a function such that $\left|f^{(10)}(x)\right| < 15$ for all x in the interval (0,1). What is the max possible error for the ninth degree Taylor polynomial centered at 0 for this function when approximating $f(1)$?