Integration WS #1-10

1. $-\frac{1}{5}\int (4-5x)^6(-5\,dx)$ u-sub

$u = 4-5x$

$du = -5\,dx$

$-\frac{1}{5}\int u^6\,du = -\frac{1}{5}\cdot\frac{u^7}{7} + C$

$= -\frac{1}{35}(4-5x)^7 + C$

2. $\int \frac{5-e^x}{e^{2x}}\,dx = \int \frac{5}{e^{2x}} - \frac{e^x}{e^{2x}}\,dx$

$= \int 5e^{-2x}\,dx - \int e^{-x}\,dx$

$= -\frac{5}{2}e^{-2x} + e^{-x} + C$

$\int e^{ax}\,dx$ $\quad u = ax$, $du = a\,dx$

$= \frac{1}{a}e^{ax} + C$

3. $-\int \frac{-1\,dx}{\sqrt{x}\,\underbrace{(1-2\sqrt{x})}}$

$\star\ \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}dx$

$u = 1-2\sqrt{x}$

$du = \frac{-1}{\sqrt{x}}dx$

$-\int \frac{du}{u}$

$= -\ln|u| + C$

$= -\ln|1-2\sqrt{x}| + C$

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4. $\int \frac{3x}{x-5}\,dx$

$$\begin{array}{r|l} & 3 \\ x-5 & 3x \\ & -3x+15 \\ \hline & 15 \end{array}$$

$\int 3 + \frac{15}{x-5}\,dx$

$3x + 15\ln|x-5| + C$

5. $\int \sqrt{x-x^3}\,(9x^2-3)\,dx$

$9x^2-3 = -(3-9x^2)$

$-3(1-3x^2)$

$u = x - x^3$

$du = 1 - 3x^2$

$$\int \underbrace{\sqrt{x-x^3}}_{u}\,(-3)\underbrace{(1-3x^2)dx}_{du}$$

$$-3\int \sqrt{u}\,du = -3\int u^{1/2}\,du$$

$$= -\cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2} + C$$

$$= -2\,(x-x^3)^{3/2} + C$$

6. $\displaystyle\int \frac{6e^{-4x}}{2+e^{-4x}}\,dx$ $\qquad \displaystyle\frac{6}{-4}\int \frac{(-4e^{-4x}\,dx)}{2+e^{-4x}}$

$u = 2 + e^{-4x}$

$du = -4e^{-4x}\,dx$

$$-\frac{3}{2}\int \frac{1}{u}\,du = -\frac{3}{2}\ln|u| + C$$

$$-\frac{3}{2}\ln\left(2+e^{-4x}\right) + C$$

7. $\displaystyle\int (1-3x^2)^2\,dx = \int (1-6x^2+9x^4)\,dx$

$$= x - 2x^3 + \frac{9}{5}x^5 + C$$

8. $\int (x^{\pi} + \underline{e^{\pi}})\,dx$

$\overline{\text{const}}$

$$= \frac{x^{\pi+1}}{\pi+1} + e^{\pi}x + C$$

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9. $\displaystyle\int \frac{x^2+x-1}{x^2+1}\,dx$

long division:

$$\begin{array}{r|l} & \quad 1 \\ x^2+1 & \overline{\;x^2+x-1\;} \\ & -\;x^2 \quad\;\; \overset{-}{+}1 \\ \hline & \qquad\quad x-2 \end{array}$$

$$= \int 1 + \frac{x-2}{x^2+1}\,dx$$

$$= \int 1 + \frac{1}{2}\frac{2x}{x^2+1} + \frac{-2}{x^2+1}\,dx$$

$$= x + \frac{1}{2}\ln(x^2+1) - 2\arctan x + C$$

OR: $\displaystyle\int \frac{x^2+x-1}{x^2+1}\,dx$

$$\int \frac{(x^2+1)+x-2}{(x^2+1)}\,dx$$

$$= \int 1 + \frac{x}{x^2+1} - \frac{2}{x^2+1}\,dx$$

$$= x + \frac{1}{2}\ln(x^2+1) - 2\arctan x + C$$

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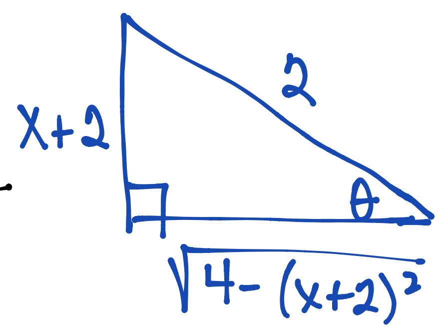
10. $\displaystyle\int \frac{dx}{\sqrt{-x^2-4x}}$

$$\left.\begin{matrix}\sqrt{x^2+a^2}\\ \sqrt{x^2-a^2}\\ \sqrt{a^2-x^2}\end{matrix}\right\}?$$

$x = a\sin\theta$

$$= \int \frac{dx}{\sqrt{4-(x^2+4x+4)}}$$

$$= \int \frac{dx}{\sqrt{4-(x+2)^2}} = \int \frac{du}{\sqrt{a^2-u^2}}$$



$$\Rightarrow x + 2 = 2\sin\theta \qquad \frac{\sqrt{4-(x+2)^2}}{2} = \frac{a}{H} = \cos\theta$$

$$x = 2\sin\theta - 2$$

$$dx = 2\cos\theta \, d\theta \qquad \sqrt{4-(x+2)^2} = 2\cos\theta$$

$$\int \frac{2\cos\theta \, d\theta}{2\cos\theta} = \int d\theta = \theta + C$$

$$= \boxed{\sin^{-1}\left(\frac{x+2}{2}\right) + C}$$

$$\frac{x+2}{2} = \sin\theta$$

$$\sin^{-1}\left(\frac{x+2}{2}\right) = \theta$$