Int. WLS #11-20

11) $\frac{1}{3}\int \frac{3dx}{2+9x^2}$ $\quad a=\sqrt{2}$

$u = 3x$

$du = 3dx$

$$\frac{1}{3}\cdot\frac{1}{\sqrt{2}}\arctan\frac{3x}{\sqrt{2}} + C$$

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12) $\int \frac{1-\ln x}{x}dx = \int \frac{1}{x} - \frac{\ln x}{x}dx$

$\int \frac{1}{x}dx - \int \frac{\ln x}{x}dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$, $\int u\,du = \frac{u^2}{2}$

$\ln|x| - \frac{(\ln x)^2}{2} + C$

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13) $\frac{1}{4\pi}\int 4\pi x \sin(2\pi x^2)\,dx$

$u = 2\pi x^2$

$du = 4\pi x\,dx$

$$\frac{1}{4\pi}\int \sin u\,du$$

$$= -\frac{1}{4\pi}\cos u + C$$

$$\boxed{-\frac{1}{4\pi}\cos(2\pi x^2) + C}$$

14) $-\frac{1}{2}\int \frac{-2(x+2)}{\sqrt{-x^2-4x}}\,dx$

$u = -x^2 - 4x$

$du = (-2x-4)\,dx$

$= -2(x+2)\,dx$

$$-\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\cdot 2u^{1/2} + C$$

$$\boxed{-\sqrt{-x^2-4x} + C}$$

15) $\int \frac{\cos(1/x)}{x^2}\,dx = -\int \cos(1/x)\cdot -1/x^2\,dx$

$-\int \cos u\,du$ $\qquad u = 1/x$

$du = -1/x^2\,dx$

$= -\sin u + C$

$-\sin(1/x) + C$

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16) $\int \underset{A}{x}\,\underset{E}{e^{-x}}\,dx$ $\qquad u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$-xe^{-x} + \int +e^{-x}\,dx$

$-xe^{-x} - e^{-x} + C$

17) $\int \frac{e^{1/x}}{x^2}\,dx = -\int e^{1/x} \cdot -\frac{1}{x^2}\,dx$

$u = 1/x \quad du = -1/x^2\,dx$

$-\int e^u\,du = -e^u + C$

$-e^{1/x} + C$

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18) $\int e^x \sin x\,dx$

$u = \sin x \quad dv = e^x dx$

$du = \cos x\,dx \rightarrow v = e^x$

$= e^x \sin x - \int e^x \cos x\,dx$

$u = \cos x \quad dv = e^x dx$

$du = -\sin x\,dx \rightarrow v = e^x$

$= e^x \sin x - \left[e^x \cos x - \int e^x(-\sin x)\,dx \right]$

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

$+ \int e^x \sin x\,dx \qquad + \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + C$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x \sin x - \frac{1}{2}e^x \cos x + C$$

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19) $\frac{1}{4}\int 4x^3 e^{x^4}\,dx$ $\quad u = x^4 \quad du = 4x^3 dx$

$$\frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$$

$$\frac{1}{4}e^{x^4} + C$$

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20) $\int (\ln x)^2\,dx$ $\quad u = (\ln x)^2 \quad dv = dx$

$du = 2(\ln x)\frac{1}{x}dx \rightarrow v = x$

$$= x(\ln x)^2 - \int 2\ln x \cdot \frac{1}{x} \cdot x\,dx$$

$$= x(\ln x)^2 - 2\int \ln x\,dx$$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \rightarrow v = x$

$$= x(\ln x)^2 - 2\left(x\ln x - \int x \cdot \frac{1}{x}\,dx\right)$$

$$= x(\ln x)^2 - 2(x \ln x - x) + C$$

$$= x(\ln x)^2 - 2x \ln x + 2x + C$$