Integration w/s #21-30

21) $\int \arccos(x)\,dx$ $\quad u = \cos^{-1}(x)$, $dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$, $v = x$

$= x\cos^{-1}(x) + \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$, $du = -2x\,dx$

$-\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = \frac{-\frac{1}{2}u^{1/2}}{1/2}$

$= x\cos^{-1}(x) - \sqrt{1-x^2} + C$

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22) $\int \frac{\ln x}{x^2}\,dx$ $\quad u = \ln x$, $dv = x^{-2}\,dx$

$du = \frac{1}{x}\,dx$, $v = -x^{-1}$

$-\frac{\ln x}{x} - \int \frac{-1}{x}\cdot\frac{1}{x}\,dx = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$

$= -\frac{\ln x}{x} - \frac{1}{x} + C$

23) $\int_0^1 \ln(1+x^2)\,dx$

$u = \ln(1+x^2)$, $dv = dx$

$du = \frac{2x}{1+x^2}\,dx$, $v = x$

$$= x\ln(1+x^2)\Big|_0^1 - \int_0^1 \frac{2x^2}{1+x^2}\,dx$$

$$1+x^2 \;\overline{)\,2x^2}\;\;2$$
$$-2x^2+2$$
$$-2$$

$$= x\ln(1+x^2)\Big|_0^1 - \int_0^1 2 - \frac{2}{1+x^2}\,dx$$

$$= \left[x\ln(1+x^2) - 2x + 2\arctan(x)\right]_0^1$$

$$= \ln 2 - 2 + 2(\pi/4) - (0 - 0 + 0)$$

$$= \ln 2 - 2 + \pi/2$$

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24) $\int x^3\cos(2x)\,dx$

	u	dv
+	x^3	$\cos(2x)$
−	$3x^2$	$\frac{1}{2}\sin(2x)$
+	$6x$	$-\frac{1}{4}\cos(2x)$
−	6	$-\frac{1}{8}\sin(2x)$
+	0	$\frac{1}{16}\cos(2x)$

$$\frac{1}{2}x^3\sin(2x) + \frac{3x^2}{4}\cos(2x) - \frac{3x}{4}\sin(2x) - \frac{3}{8}\cos(2x) + C$$

25) $\int x^2(x-2)^{3/2}\,dx$ $\quad u = x-2 \quad du = dx$

$\int (u+2)^2 u^{3/2}\,du \quad x = u+2$

$= \int (u^2+4u+4)\,u^{3/2}\,du = \int u^{7/2} + 4u^{5/2} + 4u^{3/2}\,du$

$= \frac{2}{9}u^{9/2} + 4\cdot\frac{2}{7}u^{7/2} + 4\cdot\frac{2}{5}u^{5/2} + C$

$= \frac{2}{9}(x-2)^{9/2} + \frac{8}{7}(x-2)^{7/2} + \frac{8}{5}(x-2)^{5/2} + C$

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26) $\frac{dy}{dx} = \ln x \rightarrow \int dy = \int \ln x\,dx$

$y = x\ln x - x + C$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$x\ln x - \int \frac{1}{x}\cdot x\,dx$

$x\ln x - x + C$

27) $\cos(y)\cdot\frac{dy}{dx} = 2x$

$$\int \cos(y)\,dy = \int 2x\,dx$$

$$\sin y = x^2 + C$$

or $y = \arcsin(x^2 + C)$

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28) $\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx$

$$-\int (1-\cos^2 x)(-\sin x\,dx)$$

$u = \cos x$

$du = -\sin x\,dx$

$$-\int (1-u^2)\,du = -u + u^3/3 + C$$

$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

29) $\int \sec^2(3x)\,dx = \frac{1}{3}\tan(3x) + C$

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30) $\frac{2}{\pi}\int \cos^2\left(\frac{\pi x}{2}\right)\frac{\pi}{2}\,dx \qquad u = \frac{\pi x}{2} \qquad du = \frac{\pi}{2}\,dx$

$$\frac{2}{\pi}\int \cos^2 u\,du = \frac{2}{\pi}\left(\frac{1}{2}u + \frac{1}{2}\sin u\cos u + C\right)$$

$$\frac{2}{\pi}\left(\frac{1}{2}\left(\frac{\pi x}{2}\right) + \frac{1}{2}\sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi x}{2}\right) + C\right)$$

$$\frac{x}{2} + \frac{1}{\pi}\sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi x}{2}\right) + C$$