Integration WS # 31-40

31) $\int \tan\theta \sec^6\theta \, d\theta = \int \sec^5\theta \cdot \underbrace{\sec\theta \tan\theta \, d\theta}_{du}$

$u = \sec\theta$

$\int u^5 \, du = \frac{u^6}{6} + C$

$\frac{1}{6}\sec^6(\theta) + C$

32) $\int (\sin(x) + \cos(x))^2 dx$

$\int (\sin^2(x) + \underbrace{2\sin(x)\cos(x)}_{= \sin(2x)} + \cos^2(x)) \, dx$

$\int (\sin(2x) + 1) \, dx$

$-\frac{1}{2}\cos(2x) + x + C$

33) $\int \sec^5(x)\tan(x)\,dx \;= \int \sec^4(x)\cdot \sec(x)\tan(x)\,dx$

$u = \sec(x) \quad du = \sec x \tan x\,dx$

$\int u^4\,du = \frac{1}{5}u^5 + C$

$\frac{1}{5}\sec^5(x) + C$

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34) $\int x\sin^2(x)\,dx$ IBP

$u = x \quad dv = \sin^2(x)\,dx$

☆ $\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x) + C$

$du = dx \qquad v = \frac{1}{2}x - \frac{1}{2}\sin x\cos x$

$x\left(\frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x)\right) - \int \underbrace{\frac{1}{2}x}\; - \underbrace{\frac{1}{2}\cdot\frac{1}{2}\,2\sin x\cos x}\,dx$

$2\sin x\cos x = \sin(2x)$

$\frac{1}{2}x^2 - \frac{1}{2}x\sin(x)\cos(x) - \frac{1}{4}x^2 + \int \frac{1}{4}(\sin(2x))\,dx$

$\frac{1}{4}x^2 - \frac{1}{2}x\sin(x)\cos(x) - \frac{1}{8}\cos(2x) + C$

$$35)\ \int \sin^3(x)\cos^4(x)\,dx = \int \underbrace{\underline{\sin^2(x)}\cos^4(x)}_{\text{want all in }\cos(x)}\ \ \underset{\substack{\uparrow\\ du}}{\underline{\sin(x)dx}}$$

$$= \int \left(1-\cos^2(x)\right)\cos^4(x)\ \ \sin(x)\,dx$$

$$= -\int \left(\cos^4(x)-\cos^6(x)\right)\left(-\sin(x)\,dx\right)$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$-\int u^4 - u^6\ du$$

$$-\frac{1}{5}u^5 + \frac{1}{7}u^7 + C$$

$$= -\frac{1}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C$$

36) $\displaystyle\int_{\pi/8}^{\pi/4} \sin^2(2\theta)\, d\theta$

$$\frac{1}{2}\left(\frac{1}{2}(2\theta) - \frac{1}{2}\sin(2\theta)\cos(2\theta)\right)\Bigg|_{\pi/8}^{\pi/4}$$

$$\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\cos(2\theta)\Bigg|_{\pi/8}^{\pi/4}$$

$$\left(\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\underset{1}{\sin\left(\frac{\pi}{2}\right)}\underset{0}{\cos\left(\frac{\pi}{2}\right)}\right) - \left(\frac{1}{2}\left(\frac{\pi}{8}\right) - \frac{1}{4}\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)\right)$$

$$\frac{\pi}{8} - \frac{\pi}{16} + \frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\pi}{16} + \frac{1}{8} \quad \text{or} \quad \frac{\pi + 2}{16}$$

☆ $\displaystyle\int \cos(a \cdot x)\, dx = \frac{1}{a}\sin(ax) + C$

37) $\int_0^{\pi/4} \tan^3 x \, dx = \int_0^{\pi/4} \tan x \cdot \underline{\tan^2 x} \, dx$

$= \int_0^{\pi/4} \tan x \, (\sec^2 x - 1) \, dx$

$= \int_0^{\pi/4} \tan x \sec^2 x \, dx - \int_0^{\pi/4} \tan x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int_0^1 u \, du$

$\frac{u^2}{2} \Big|_0^1 = \boxed{\frac{1}{2}} + \ln|\cos x| \Big|_0^{\pi/4}$

$\frac{1}{2} + \left[\ln|\cos \pi/4| - \underbrace{\ln|\cos 0|}_{\substack{=1 \\ \ln 1 = 0}} \right]$

$\boxed{\frac{1}{2} + \ln\left(\frac{\sqrt{2}}{2}\right)}$

38) $$\int \frac{x-3}{x^2+4x+3}\,dx$$

$(x+3)(x+1)$ PFD

$$\frac{A}{x+3} + \frac{B}{x+1} = \frac{x-3}{(x+3)(x+1)}$$

$$A(x+1) + B(x+3) = x-3$$

$x=-1:$ $0 + 2B = -4 \rightarrow B = -2$

$x=-3:$ $-2A + 0 = -6 \rightarrow A = 3$

$$\int \frac{3}{x+3} - \frac{2}{x+1}\,dx = 3\ln|x+3| - 2\ln|x+1| + C$$

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39) $$\int \frac{1-x}{2x^2+x}\,dx \qquad \frac{A}{x} + \frac{B}{2x+1}$$

$x(2x+1)$

$$A(2x+1) + Bx = 1-x$$

$x=-1/2:$ $0 - \frac{1}{2}B = \frac{3}{2} \rightarrow B = -3$

$x=0:$ $A = 1$

$$\int \frac{1}{x} - \frac{3}{2x+1}\,dx$$

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}\int \frac{3}{u}\,du$

$\frac{3}{2}\ln|u|$

$$= \boxed{\ln|x| - \frac{3}{2}\ln|2x+1| + C}$$

40) $$\int \frac{x^2+1}{x^3+3x-4}\,dx$$

$u = x^3+3x-4$

$du = (3x^2+3)\,dx$

$\frac{1}{3}\,du = (x^2+1)\,dx$

$$\frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$$

$$\frac{1}{3}\ln|x^3+3x-4| + C$$