Integration WS #41-50

41) $\int 2x\sqrt{2x-3}\,dx$

$u = 2x-3 \leftarrow x = \left(\frac{u+3}{2}\right)$

$du = 2\,dx$

$\int (x)\sqrt{u}\,du$

$\int \left(\frac{u+3}{2}\right) u^{1/2}\,du = \frac{1}{2}\int \left(u^{3/2} + 3u^{1/2}\right) du$

$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} + \cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2}\right) + C$

$= \frac{1}{5}(2x-3)^{5/2} + (2x-3)^{3/2} + C$

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42) $\int x^2 \ln x\,dx$

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \int \frac{x^{\cancel{3}\,2}}{3}\cdot\frac{1}{\cancel{x}}\,dx$

$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

43) $\int \cos^3(2\theta)\,d\theta = \int \underbrace{\cos^2(2\theta)}\cdot \cos(2\theta)\,d\theta$

$$= \frac{1}{2}\int (1-\sin^2(2\theta))\,2\cos(2\theta)\,d\theta$$

$u = \sin(2\theta) \quad du = 2\cos(2\theta)\,d\theta$

$$\frac{1}{2}\int (1-u^2)\,du = \frac{1}{2}\left(u - \frac{u^3}{3}\right) + C$$

$$= \frac{1}{2}\sin(2\theta) - \frac{1}{6}\sin^3(2\theta) + C$$

44) $-\frac{1}{3}\int \frac{-3\cdot 2x}{\sqrt{9-3x^2}}\,dx \qquad u = 9-3x^2$

$du = -6x\,dx$

$$-\frac{1}{3}\int \frac{du}{\sqrt{u}} = -\frac{1}{3}\int u^{-1/2}\,du = -\frac{1}{3}\cdot 2\,u^{1/2} + C$$

$$-\frac{2}{3}(9-3x^2)^{1/2} + C$$

45) $\int \frac{2\sqrt{x}\,e^{\sqrt{x}}}{2\sqrt{x}}\,dx = \int 2\sqrt{x}\cdot\frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$u = 2\sqrt{x} \qquad dv = \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$du = \frac{1}{\sqrt{x}}\,dx \qquad v = e^{\sqrt{x}}$

$$= 2\sqrt{x}\, e^{\sqrt{x}} - 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\, dx$$

$$= 2\sqrt{x}\, e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

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46) $\displaystyle\int \frac{1 + \sin(e^{-2x})}{e^{2x}}\, dx$

$$= -\frac{1}{2}\int \left(1 + \sin(e^{-2x})\right) \cdot -2e^{-2x}\, dx \qquad u = e^{-2x}$$

$$du = -2e^{-2x} dx$$

$$-\frac{1}{2}\int (1 + \sin u)\, du$$

$$-\frac{1}{2} u + \frac{1}{2}\cos u + C$$

$$= -\tfrac{1}{2} e^{-2x} + \tfrac{1}{2}\cos(e^{-2x}) + C$$

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47) $\displaystyle\int \frac{x-1}{x^2 - 3x}\, dx$

$x(x-3)$

$$\frac{A}{x} + \frac{B}{x-3} = \frac{x-1}{x(x-3)}$$

$$A(x-3) + Bx = x - 1$$

$x=3$: $3B = 2 \Rightarrow B = 2/3$

$x=0$: $-3A = -1 \Rightarrow A = 1/3$

$$\int \frac{1/3}{x} + \frac{2/3}{x-3}\,dx = \underline{\frac{1}{3}\ln|x| + \frac{2}{3}\ln|x-3| + C}$$

48) $$\int \frac{x^2}{x^2+2x-15}\,dx$$

$$x^2+2x-15 \,\overline{\big)\, x^2}^{\,1}$$
$$\underline{-x^2 \mp 2x + 15}$$
$$-2x+15$$

$$= \int 1 + \frac{-2x+15}{(x+5)(x-3)}\,dx = \int dx + \underbrace{\int \frac{-2x+15}{(x+5)(x-3)}\,dx}$$

$$\downarrow$$

$$x + \boxed{\text{PFD}} + C$$

$$\frac{A}{x+5} + \frac{B}{x-3} = \frac{-2x+15}{(x+5)(x-3)}$$

$$A(x-3) + B(x+5) = -2x+15$$

$x=3$: $\quad 8B = 9 \rightarrow B = 9/8$

$x=-5$: $\quad -8A = 25 \rightarrow A = -25/8$

$$\int \frac{-25/8}{x+5} + \frac{9/8}{x-3}\,dx = \underline{-\frac{25}{8}\ln|x+5| + \frac{9}{8}\ln|x-3|} + C$$

$$\underline{= x - \frac{25}{8}\ln|x+5| + \frac{9}{8}\ln|x-3| + C}$$

49) $\int \frac{t}{\sqrt{1-t^4}}\,dt = \frac{1}{2}\int \frac{2t\,dt}{\sqrt{1-(t^2)^2}}$

$u = t^2 \quad du = 2t\,dt$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\arcsin u + C$

$= \frac{1}{2}\arcsin(t^2) + C$

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50) $\int \frac{1}{x^{2/3}(1+x^{1/3})}\,dx$

$u = 1 + x^{1/3}$

$du = \frac{1}{3}x^{-2/3}\,dx$

$= 3\int \frac{1}{(1+x^{1/3})}\underbrace{\frac{1}{3}x^{-2/3}\,dx}$

$3\int \frac{1}{u}\,du$

$3\ln|u| + C$

$= 3\ln|1+x^{1/3}| + C$