Integration WS 51-60

51) $\int \frac{3x^3-2}{x^2+4}\,dx$

$$\begin{array}{r} 3x \qquad\qquad \\ x^2+4 \,\big)\overline{3x^3 \qquad\quad -2} \\ -\;3x^3+12x \qquad \\ \hline -12x-2 \end{array}$$

$= \int 3x - \frac{12x+2}{x^2+4}\,dx$

$= \int 3x - \frac{\overset{6\cdot 2}{\cancel{12}}x}{x^2+4} - \frac{2}{x^2+4}\,dx$

$u = x^2+4$

$du = 2x\,dx$

$6\int \frac{du}{u}$

$-2\int \frac{1}{2^2+x^2}\,dx$

arctan → $u = x$

$a = 2$

$\frac{3x^2}{2} - 6\ln|\underline{\underline{u}}| - 2\cdot\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

$\frac{3x^2}{2} - 6\ln(x^2+4) - \arctan\left(\frac{x}{2}\right) + C$

---

52) $\int \frac{1}{\sqrt{-x^2+2x}}\,dx = \int \frac{1}{\sqrt{1-(x^2-2x+1)}}\,dx$

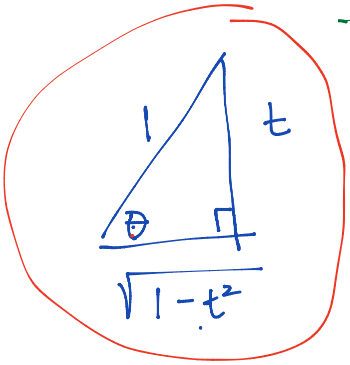
$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right) + C$

$$\int \frac{1}{\sqrt{1-(x-1)^2}}\,dx = \arcsin(x-1) + C$$

---

53) $$\int \frac{t^2}{(1-t^2)^{3/2}}\,dt = \int \frac{t^2}{\left(\sqrt{1-t^2}\right)^3}\,dt$$

$\sqrt{1-t^2}$



$$\boxed{t = \text{sen}\,\theta}$$

$$dt = \cos\theta\, d\theta$$

$$\sqrt{1-t^2} = \cos\theta$$

$$\int \frac{\sin^2\theta \cdot \cos\theta\, d\theta}{\cos^3\theta}$$

$$\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \int \tan^2\theta\, d\theta$$

$$= \int (\sec^2\theta - 1)\,d\theta = \underline{\tan\theta - \theta + C}$$

$$= \frac{t}{\sqrt{1-t^2}} - \arcsin(t) + C$$

---

54) $\int \sin^3 \overset{\swarrow}{x} \cdot \cos^2 x \, dx = \int \underbrace{\boxed{\sin^2 x}\cos^2 x}_{(1-\cos^2 x)} \quad \underline{\sin x\, dx}$

$= -\int (\cos^2 x - \cos^4 x) \, -\sin x \, dx$

$\underline{u = \cos x} \quad du = -\sin x\, dx$

$-\int (u^2 - u^4)\, du = -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$

$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

---

55) $\int \frac{2x-3}{\sqrt{9-x^2}}\, dx = -\int \frac{-2x}{\sqrt{9-x^2}}\, dx - \int \frac{\boxed{3}\ 1}{\sqrt{9-x^2}}\, dx$

$u = 9 - x^2$

$du = -2x\, dx$

$-\int u^{-1/2}\, du$

$= -2u^{1/2} + C$

$a = 3$

$\arcsin \frac{u}{a}$

$3\arcsin\frac{x}{3}$

$= -2\sqrt{9-x^2} + 3\arcsin\left(\frac{x}{3}\right) + C$

56) $\int \ln x^3 \, dx = \int 3 \ln x \, dx$

$u = 3 \ln x \qquad dv = dx$

$du = \frac{3}{x} dx \longrightarrow v = x$

$= 3x \ln x - \int \cancel{x} \cdot \frac{3}{\cancel{x}} \, dx$

$= 3x \ln x - 3x + C$

57) $\int \frac{2}{\sqrt{3 + 2x - x^2}} \, dx \qquad \int \frac{1}{\sqrt{a^2 - u^2}} \, du$

$2 \int \frac{1}{\sqrt{4 - (x^2 - 2x + 1)}} \, dx = 2 \int \frac{1}{\sqrt{2^2 - (x-1)^2}} \, dx$

$a = 2 \quad u = x - 1$

$= 2 \cdot \operatorname{arcsen}\left(\frac{x-1}{2}\right) + C$

58) $-\frac{1}{3}\int \frac{-3(t^2-3)}{-t^3+9t-1}\,dt$

$u = -t^3+9t-1$

$du = (-3t^2+9)\,dt$

$= -3(t^2-3)\,dt$

$-\frac{1}{3}\int \frac{du}{u} = \frac{-1}{3}\ln|u|+C$

$= \frac{-1}{3}\ln|-t^3+9t-1| + C$

---

59) $\int \tan^2(2x)\,dx = \int(\sec^2(2x)-1)\,dx$

$\frac{1}{2}\tan(2x) - x + C$

---

60) $\int \frac{x^2}{x^2-3x+2}\,dx$

$$\begin{array}{r} 1 \\ x^2-3x+2 \overline{\big)\, x^2 } \\ -x^2+3x \overset{-}{+} 2 \\ \hline 3x-2 \end{array}$$

$= \int 1 + \frac{3x-2}{\underset{(x-2)(x-1)}{x^2-3x+2}}\,dx$

$\frac{A}{x-2} + \frac{B}{x-1} = \frac{3x-2}{(x-2)(x-1)}$

$A(x-1) + B(x-2) = 3x-2$

$x=1:\quad -B = 1 \rightarrow B = -1$

$x=2:\quad A = 4$

$$\int 1 + \frac{4}{x-2} - \frac{1}{x-1}\,dx$$

$$= x + 4\ln|x-2| - \ln|x-1| + C$$