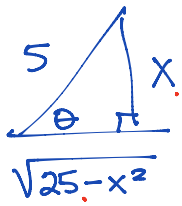
Integration WS #61-70

61) $\int \frac{2e^{1/x}}{x^2}dx = -2\int e^{1/x}\cdot -1/x^2\,dx$ $\quad u = 1/x$, $du = -1/x^2\,dx$

$-2\int e^u du = -2e^u + C$

$= -2e^{1/x} + C$

62) $\int \frac{1}{x^2\sqrt{25-x^2}}dx$



$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$

$\sqrt{25-x^2} = 5\cos\theta$

$\int \frac{5\cos\theta\,d\theta}{25\sin^2\theta\cdot 5\cos\theta} = \frac{1}{25}\int \frac{1}{\sin^2\theta}d\theta$

$= \frac{1}{25}\int \csc^2\theta\,d\theta$

$= -\frac{1}{25}\cot\theta + C$

$= -\frac{1}{25}\left(\frac{\sqrt{25-x^2}}{x}\right) + C$

63) $\int \frac{3e^x - 3e^{-x}}{(e^x+e^{-x})^2}dx$ $\quad u = e^x + e^{-x}$

$du = (e^x - e^{-x})\,dx$

$\int \frac{3(e^x-e^{-x})\,dx}{(e^x+e^{-x})^2} = \int \frac{3\,du}{u^2} = 3\int u^{-2}du$

$$-3\frac{u^{-1}}{-1}+C=\frac{-3}{u}+C$$

$$=\frac{-3}{e^x+e^{-x}}+C$$

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64) $\int \frac{x+1}{x^3+x}\,dx$

$x(x^2+1)$

$$\frac{A}{x}+\frac{Bx+C}{x^2+1}$$

$$A(x^2+1)+(Bx+C)x=x+1$$

$x=0$ $\quad A=1$

$x=1$ $\quad 2(1)+B+C=2$

$$B+C=0$$

$x=-1$ $\quad 2(1)+(-B+C)(-1)=0$

$$B-C=-2$$

$$2B=-2$$

$$B=-1$$

$$C=1$$

$$=\int \frac{1}{x}+\frac{-x+1}{x^2+1}\,dx=\int \frac{1}{x}-\frac{1}{2}\frac{2x}{x^2+1}+\frac{1}{x^2+1}\,dx$$

$$=\ln|x|-\frac{1}{2}\ln(x^2+1)+\arctan(x)+C$$

65) $\int \left(\frac{4}{x^2} - \frac{3}{x^3}\right) dx = \int (4x^{-2} - 3x^{-3})\, dx$

$= \frac{4x^{-1}}{-1} - \frac{3x^{-2}}{-2} + C = \frac{-4}{x} + \frac{3}{2x^2} + C$

66) $\int \tan^4 x \sec^2 x \, dx$

$u = \tan(x) \quad du = \sec^2(x)\, dx \qquad \int u^4 du$

$\frac{1}{5} u^5 + C$

$= \frac{1}{5} \tan^5(x) + C$

67) $\int \frac{x^3}{x^2+1} dx = \int x - \frac{1}{2} \frac{2x}{x^2+1} dx$

$= \frac{x^2}{2} - \frac{1}{2} \ln(x^2+1) + C$

68) $\int \frac{x}{x^4+9} dx = \frac{1}{2} \int \frac{2x}{(x^2)^2 + 3^2} dx$

$u = x^2 \quad a = 3$

$du = 2x\,dx$

$\frac{1}{2} \int \frac{du}{u^2+a^2} = \frac{1}{2} \cdot \frac{1}{a} \arctan\frac{u}{a} + C$

$= \frac{1}{6} \arctan\left(\frac{x^2}{3}\right) + C$

69) $\int \cos\theta \cdot \sin(\sin\theta)\, d\theta$ $\quad u = \sin\theta$, $du = \cos\theta\, d\theta$

$$\int \sin(u)\, du = -\cos(u) + C$$

$$= -\cos(\sin\theta) + C$$

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70) $\int \frac{2(\ln x)^2}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}\, dx$

$$\int 2u^2\, du = 2\,\frac{u^3}{3} + C$$

$$\frac{2}{3}(\ln x)^3 + C$$