

Integration WS # 71-80

$$71) \frac{-1}{2} \int \frac{-2(x+1)}{(3-2x-x^2)^3} dx$$

$$u = 3 - 2x - x^2 \\ du = (-2 - 2x) dx \\ = -2(1+x) dx$$

$$-\frac{1}{2} \int \frac{1}{u^3} du$$

$$= -\frac{1}{2} \int u^{-3} du = -\frac{1}{2} \frac{u^{-2}}{-2} + C$$

$$= \frac{1}{4} (3-2x-x^2)^{-2} + C$$

$$72) \int (2x-1) e^{(x^2-x)} dx \quad u = x^2 - x \quad du = (2x-1) dx \\ \int e^u du = e^u + C$$

$$= e^{x^2-x} + C$$

$$73) \int \frac{x}{x^2+4x+8} dx$$

$$u = x^2 + 4x + 8 \\ du = (2x+4) dx \\ 2(x+2) dx$$

$$= \int \frac{x+2-2}{x^2+4x+8} dx = \frac{1}{2} \int \frac{2(x+2)}{x^2+4x+8} dx - \int \frac{2 dx}{x^2+4x+8}$$

$$\frac{1}{2} \int \frac{1}{u} du \quad \uparrow \\ \frac{1}{2} \ln|x^2+4x+8|$$

$$= \frac{1}{2} \ln |x^2 + 4x + 8| - \int \frac{2 dx}{(x^2 + 4x + 4) + 4}$$



$$- 2 \int \frac{1}{(x+2)^2 + 4} dx$$

$$u = x + 2$$

$$a = 2$$

$$= \frac{1}{2} \ln |x^2 + 4x + 8| - 2 \left(\frac{1}{2} \right) \arctan \left(\frac{x+2}{2} \right) + C$$

$$74) \int \frac{2t - 1}{t^2 + 9} dt = \int \frac{2t}{t^2 + 9} - \frac{1}{t^2 + 9} dt$$

$$u = t^2 + 9$$

$$du = 2t dt$$

$$\int \frac{1}{u} du$$

$$\uparrow$$

$$\arctan$$

$$\text{w/ } a = 3$$

$$u = t$$

$$= \ln(t^2 + 9) - \frac{1}{3} \arctan \left(\frac{t}{3} \right) + C$$

$$75) \int \ln(e^{x^5}) dx = \int x^5 dx = \frac{1}{6} x^6 + C$$

$$76) \int \frac{1}{\sqrt{2-4x-x^2}} dx$$

$$\int \frac{1}{\sqrt{a^2-u^2}} du = \arcsin \frac{u}{a} + C$$

$$= \int \frac{1}{\sqrt{6-(x^2+4x+4)}} dx = \int \frac{1}{\sqrt{6-(x+2)^2}} dx$$

$$a = \sqrt{6} \quad u = x+2$$

$$= \arcsin \left(\frac{x+2}{\sqrt{6}} \right) + C$$

$$77) \int \frac{(x+1)^3}{x^2} dx = \int \left(x + 3 + \frac{3}{x} + \frac{1}{x^2} \right) dx$$

$$= \frac{x^2}{2} + 3x + 3 \ln|x| - \frac{1}{x} + C$$

$$78) -\frac{1}{2} \int -2x \sqrt{4-x^2} dx$$

$$u = 4-x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \int u^{1/2} du = -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= -\frac{1}{3} (4-x^2)^{3/2} + C$$

$$79) \int_0^{\sqrt{3}/2} \frac{4}{1+x^2} dx = 4 \arctan(x) \Big|_0^{\sqrt{3}/2}$$

$$= 4 \arctan\left(\frac{\sqrt{3}}{2}\right)$$

$$80) \int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} dx$$

$$\begin{array}{r} 3x^2 + 1 \\ x^2 + x - 2 \overline{) 3x^4 + 3x^3 - 5x^2 + x - 1} \\ \underline{-3x^4 + 3x^3 + 6x^2} \\ 6x^2 - 4x - 1 \\ \underline{-6x^2 - 6x + 12} \\ -12x + 11 \\ \underline{-12x + 24} \\ 13 \end{array}$$

$$= \int \left(3x^2 + 1 + \frac{1}{(x+2)(x-1)} \right) dx$$

$$\frac{A}{x+2} + \frac{B}{x-1}$$

$$A(x-1) + B(x+2) = 1$$

$$x=1: 3B=1 \Rightarrow B = \frac{1}{3}$$

$$x=-2: -3A=1 \Rightarrow A = -\frac{1}{3}$$

$$= \int \left(3x^2 + 1 - \frac{1/3}{x+2} + \frac{1/3}{x-1} \right) dx$$

$$= x^3 + x - \frac{1}{3} \ln|x+2| + \frac{1}{3} \ln|x-1| + C$$