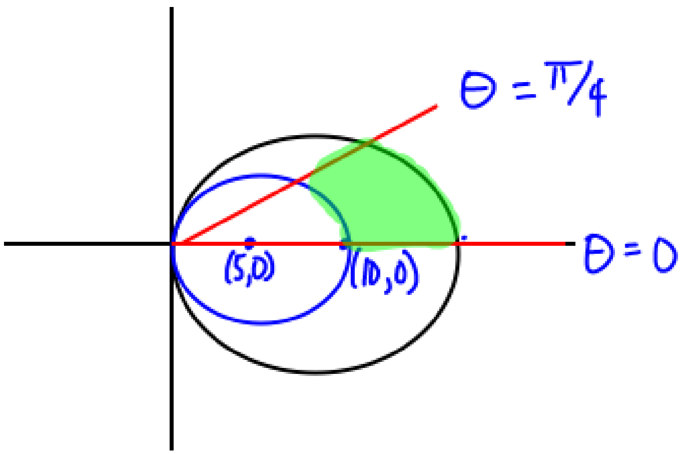
Quiz 23 Selected Questions:

3. Calculate the area of the region bounded by: $r = 20\cos(\theta)$, $r = 10\cos(\theta)$ and the rays $\theta = 0$ and $\theta = \frac{\pi}{4}$.



$$A = \frac{1}{2}\int_0^{\pi/4}\left((20\cos\theta)^2 - (10\cos\theta)^2\right)d\theta$$

$$\frac{1}{2}\int_0^{\pi/4} 300\cos^2\theta \, d\theta$$

$$150\int_0^{\pi/4}\cos^2\theta \, d\theta$$

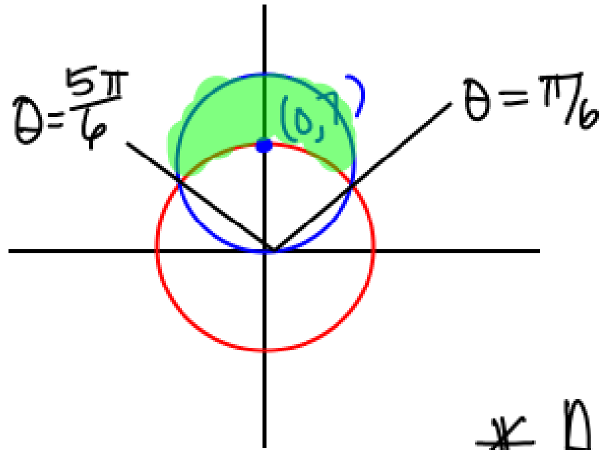
$$150\left[\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right]_0^{\pi/4}$$

$$75\theta + 75\sin\theta\cos\theta \Big|_0^{\pi/4}$$

$$\left[\frac{75\pi}{4} + 75\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\right] - [0 + 0]$$

$$\boxed{75\pi/4 + 75/2}$$

4. Which of the following integrals will represent the area outside $r = 7$, but inside $r = 14\ \sin(\theta)$?



$$7 = 14\sin\theta$$

$$\frac{1}{2} = \sin\theta$$

$$\theta = \pi/6,\ 5\pi/6$$

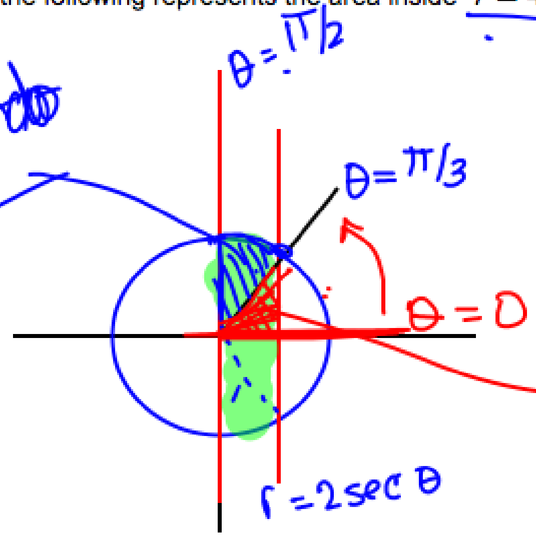
$$* \ A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left((14\sin\theta)^2 - (7)^2 \right) d\theta$$

Note :

$$A = 2\cdot\int_{\pi/6}^{\pi/2} \frac{1}{2}\left((14\sin\theta)^2 - (7)^2 \right) d\theta$$

5. Which of the following represents the area inside $r = 4$, and between the lines $\theta = \frac{\pi}{2}$ and $r = 2\sec(\theta)$?

$$\int_{\pi/3}^{\pi/2} \frac{1}{2}(4)^2\, d\theta$$



$$r = \frac{2}{\cos\theta}$$

$$r\cos\theta = 2$$

$$x = 2$$

$$\int_0^{\pi/3} \frac{1}{2}(2\sec\theta)^2\, d\theta$$

$$4 = 2\sec\theta$$

$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

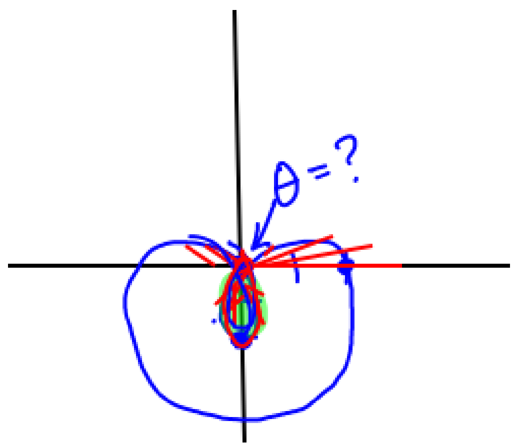
$$\theta = \pi/3$$

$$A = 2\left[\int_{\pi/3}^{\pi/2} \frac{1}{2}(4)^2\, d\theta + \int_0^{\pi/3} \frac{1}{2}(2\sec\theta)^2\, d\theta\right]$$

6. Which of the following represents the area inside the inner loop of $r = 2 - 4\sin(\theta)$?

$\theta = 0 \rightarrow r = 2 - 0 = 2$

$\theta = \pi/2 \rightarrow r = 2 - 4 = -2$



If at origin, $r = 0$

$0 = 2 - 4\sin\theta$

$\sin\theta = 1/2$

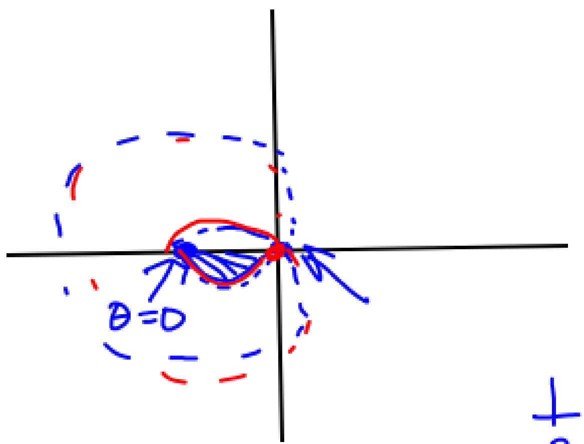
$\theta = \pi/6, \ 5\pi/6$

$$A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(2 - 4\sin\theta)^2 \, d\theta$$

extra Area of inner loop

$r = 2 - 4\cos\theta$

$\theta = 0 \to r = 2 - 4(1) = -2$



$0 = 2 - 4\cos\theta$

$\cos\theta = 1/2$

$\theta = \pi/3, 5\pi/3$

$\frac{1}{2}$ of inner loop: $\int_0^{\pi/3} \frac{1}{2}(2-4\cos\theta)^2 d\theta$

Area of inner loop: $2\int_0^{\pi/3} \frac{1}{2}(2-4\cos\theta)^2 d\theta$

OR

$$\int_0^{\pi/3} \frac{1}{2}(2-4\cos\theta)^2 d\theta + \int_{5\pi/3}^{2\pi} \frac{1}{2}(2-4\cos\theta)^2 d\theta$$

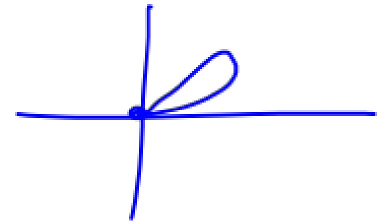
7. Which of the following represents the area inside one petal of $r = 8\,\sin(3\,\theta)$?

$0 = 8\sin(3\theta)$

$0 = \sin(3\theta)$

$3\theta = 0, \pi, 2\pi, 3\pi \ldots$

$\theta = 0, \pi/3, 2\pi/3, \pi$



$$\int_0^{\pi/3} \frac{1}{2}(8\sin(3\theta))^2\, d\theta$$

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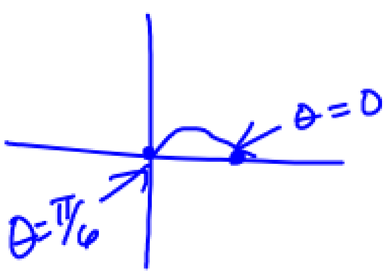
one petal of $r = 8\cos 3\theta$

$0 = 8\cos 3\theta$

$3\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

$\theta = \pi/6, \pi/2, 5\pi/6, 7\pi/6$



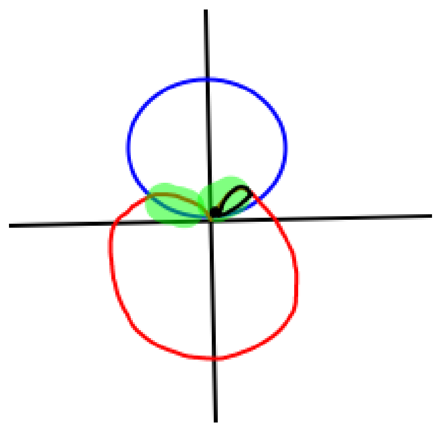
$$A = \int_{\pi/6}^{\pi/2} \frac{1}{2}(8\cos 3\theta)^2\, d\theta$$

or

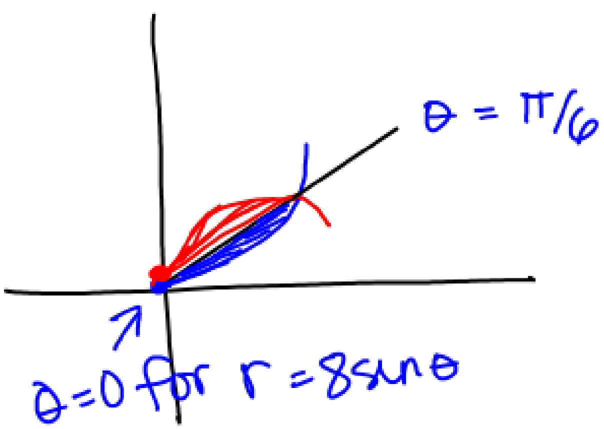
$$2\int_0^{\pi/6} \frac{1}{2}(8\cos 3\theta)^2\, d\theta$$

$0 = 8 - 8\sin\theta \rightarrow \sin\theta = 1 \rightarrow \theta = \pi/2$

8. Which of the following represents the area interior to both $r = 8 - 8\sin(\theta)$ and $r = 8\sin(\theta)$?



$$8 - 8\sin\theta = 8\sin\theta$$
$$8 = 16\sin\theta$$
$$\frac{1}{2} = \sin\theta$$
$$\theta = \pi/6$$



$$A = 2\cdot\left[\int_0^{\pi/6} \frac{1}{2}(8\sin\theta)^2\,d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}(8 - 8\sin\theta)^2\,d\theta\right]$$

10. Find the length of $r = 2\sec(\theta)$ from $\theta = 0$ to $\theta = \frac{\pi}{4}$.

$$L = \int_{\alpha}^{\beta} \sqrt{[\rho(\theta)]^2 + [\rho'(\theta)]^2}\, d\theta$$

$$\rho(\theta) = r = 2\sec\theta$$

$$\rho'(\theta) = 2\sec\theta\tan\theta$$

$$L = \int_0^{\pi/4} \sqrt{(2\sec\theta)^2 + (2\sec\theta\tan\theta)^2}\, d\theta = \int_0^{\pi/4} \sqrt{4\sec^2\theta + 4\sec^2\theta\tan^2\theta}\, d\theta$$

$$= \int_0^{\pi/4} 2\sec\theta \sqrt{\underbrace{1 + \tan^2\theta}_{\sec^2\theta}}\, d\theta = \int_0^{\pi/4} 2\sec^2\theta\, d\theta$$

$$2\tan\theta \Big|_0^{\pi/4} = \underline{2}$$