Math 2311

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Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

Math 2311
Class Notes for Section 2.2-2.4

Last week:

- Reviewed graphs
- Counting techniques
- Venn Diagrams

Popper 01

1. Which of the following lists would have the largest variance?
   a. 3, 3, 3, 3, 3
   b. 1, 2, 3, 4, 5
   c. 0, 2, 4, 6, 8
   d. 1, 3, 7, 10, 22

Some Review:

How many ways can you line 4 people up for a picture? $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 24$

How many ways can you choose 4 people from 10 for a committee? $\_\ \_\ \_\ \_$

$_{10}C_4 = 210$

$\frac{10!}{2!\,4!}$

How many ways can you arrange the letters of CASABLANCA?

75,600

C - 2 S - 1 L - 1

A - 4 B - 1 N - 1

How many ways can you get a 5 card poker hand?

$_{52}C_5$

How many ways can you get a full house 5 card poker hand?

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> 13*choose(4,3)*12*choose(4,2)
[1] 3744
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$13\left({}_4C_3\right)\cdot 12\left({}_4C_2\right)$

CAT

ACT

TAC

Popper 01

2. In how many ways can you arrange 5 people in a row?
    a. 20
    b. 120
    c. 24
    d. 5
    e. none of these

3. In how many ways can you select from 8 people, 3 to serve on a committee?
    a. 42
    b. 24
    c. 56
    d. 5040
    e. none of these

Draw a Venn Diagram for the following situation: A group of 100 people are asked about their preference for soft drinks. The results are as follows:

55 Like Coke

25 Like Diet Coke

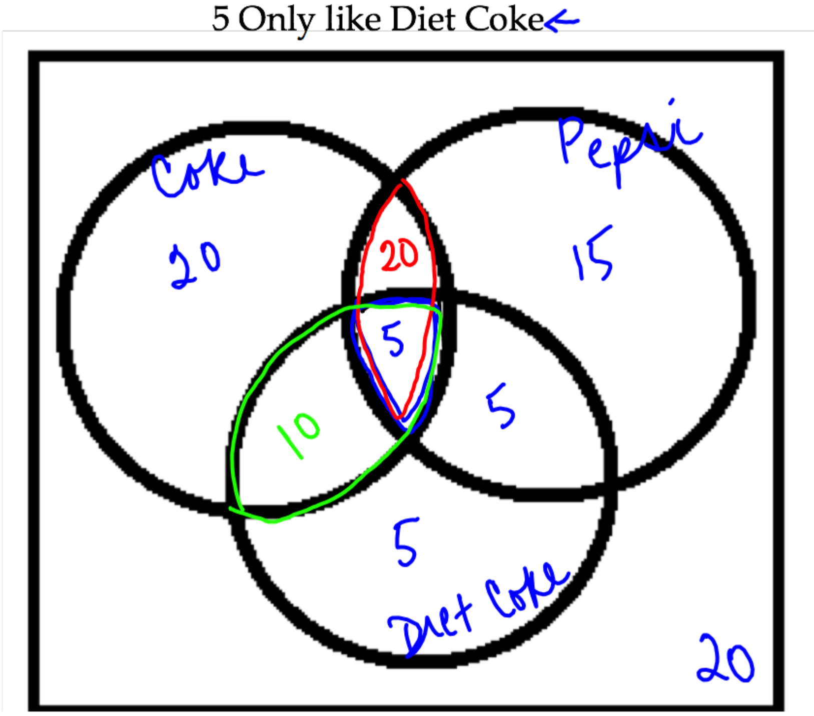
45 Like Pepsi

15 like Coke and Diet Coke

5 Like all 3 soft drinks

25 Like Coke and Pepsi

5 Only like Diet Coke



Popper 01

4. Given $A = \{a,b,d,p,z\}$ and $B = \{c,d,e,f,w,z\}$ find $A \cup B$

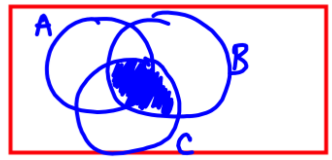
union

   a. $\{a,b,c,d,e,f,p,w,z\}$
   b. $\{d,z\}$
   c. $\{a,b,e,p,w\}$
   d. $\varnothing$
   e. none of these

intersection

5. Given $A = \{a,b,d,p,z\}$ and $B = \{c,d,e,f,w,z\}$ find $A \cap B$
   a. $\{a,b,c,d,e,f,p,w,z\}$
   b. $\{d,z\}$
   c. $\{a,b,e,p,w\}$
   d. $\varnothing$
   e. none of these

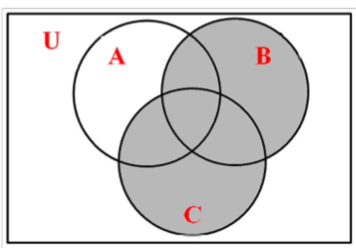
Popper 01

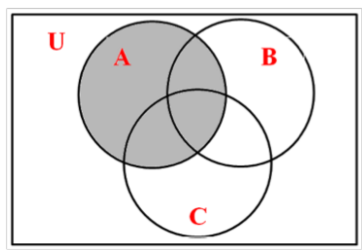


$B \cap C$

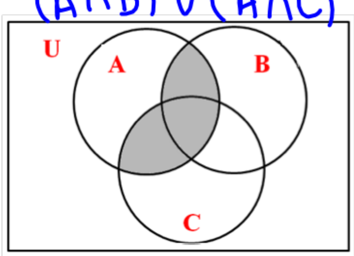
6. Which of the following is the correct Venn Diagram for $A \cup (B \cap C)$?

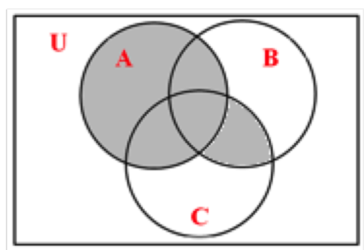
$B \cup C$ $\qquad$ A

a. 

b. 

$(A \cap B) \cup (A \cap C)$

c. 

d. 

~~e.~~ none of these

Section 2.3 – Basic Probability Models

The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

The sample space of a random phenomenon is the set of all possible outcomes.

An event is an outcome or a set of outcomes of a random phenomenon. It is a subset of the sample space. A simple event is an event consisting of exactly one outcome.

To compute the probability of some event E occurring, divide the number of ways that E can occur by the number of possible outcomes the sample space, S, can occur:

$$P(E)=\frac{n(E) \leftarrow \text{number of times event occurs}}{n(S) \leftarrow \text{total in sample space}}$$

Basic Rules of Probability

1. All events have a probability between zero and one. $0 \le P(E) \le 1$
2. All possible outcomes together must have a probability of one. $P(S) = 1$
3. Complement Rule: For any event E, $P(E^c) = 1 - P(E)$
4. Addition Rule: If A and B are disjoint events, then $\boxed{P(E \cup F) = P(E) + P(F)}$

   have nothing in common!
5. If E and F are any events of an experiment, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Disjoint: $P(E \cap F) = 0$ $(E \cap F = \emptyset)$

Examples:

1. Suppose we draw a single card from a deck of 52 fair playing cards. $n(S) = 52$

a. What is the probability of drawing a heart? $n(\text{Hearts}) = 13$ $P(\text{Heart}) = \frac{13}{52}$

b. What is the probability of drawing a queen? $n(Q) = 4$ $P(Q) = \frac{4}{52}$

14 marbles total

2. If 5 marbles are drawn at random all at once from a bag containing 8 white and 6 black marbles, what is the probability that 2 will be white and 3 will be black?

$$n(S) = {}_{14}C_5 = 2002$$

$$n(E) = {}_8C_2 \cdot {}_6C_3 = 28 \cdot 20 = 560$$

$$P(E) = \frac{560}{2002}$$

3. The qualified applicant pool for six management trainee positions consists of seven women and five men.

$$n(S) = {}_{12}C_6 = 924$$

a. What is the probability that a randomly selected trainee class will consist entirely of women?

$$n(W) = {}_{7}C_6 = 7$$

$$P(W) = 7/924$$

b. What is the probability that a randomly selected trainee class will consist of an equal number of men and women?

$$n(3W \,\&\, 3m) = {}_{7}C_3 \cdot {}_{5}C_3 = 35 \cdot 10 = 350$$

$$P(3W + 3m) = \frac{350}{924}$$

$P(S) = .48 \quad P(B) = .66 \quad P(H) = .38$

4. A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports.

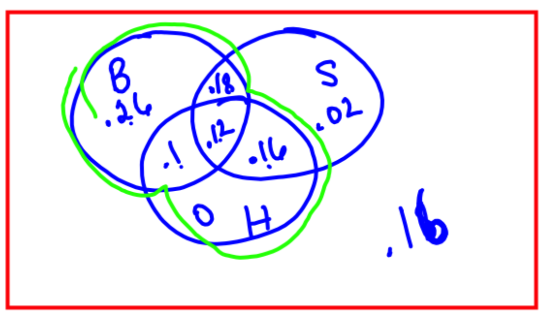
$P(S \cap B) = .3 \quad P(B \cap H) = .22 \quad P(S \cap H) = .28$

$P(S \cap B \cap H) = .12$

a. What is the probability that a randomly selected student likes basketball or hockey? Solve this by also using an appropriate formula.

$$P(B \cup H) = P(B) + P(H) - P(B \cap H)$$
$$= .66 + .38 - .22 = .82$$

b. What is the probability that a randomly selected student does not like any of these sports?



Popper 01

$P(J) = .4 \quad P(T) = .38 \quad P(M) = .22$

7. Jane, Tom, and Mary are running chairperson of the board. The probability that Jane will be chosen is 0.40. The probability that Tom will be chosen is 0.38. What is the probability that Mary will be chosen? (These are the only three people running)
   a. 0.22
   b. 0.78
   c. 0.62
   d. 0.5
   e. none of these

8. Using the above information, what is the probability that either Mary or Jane will be chosen?

   $P(M \cup J) = P(M) + P(J)$

   a. 0.22
   b. 0.78
   c. 0.62
   d. 0.5
   e. none of these

Disjoint + Mutually Exclusive

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

Not same as <u>Independence</u>

Section 2.4 – General Probability Rules

Two events are independent if knowing that one occurs does not change the probability that the other occurs.
(Note: This is not the same as sets that are disjoint or mutually exclusive)

If E and F are independent events, then $\boxed{P(E \cap F) = P(E)P(F)}$

Example:

5. If P(A) = .36 and P(B) = .58 and A and B are independent,
a. What is P(A and B)? $= (.36)(.58) = .2088$
$\cap$

b. What is the probability of A or B occurring?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .36 + .58 - .2088$$
$$= .7312$$

| given

Dependent events, the occurrence of one event does have an effect on the occurrence of the other event. The probability $P(E \mid F)$ is read "the probability of event *E* given event *F* had already occurred". If *E* and *F* are independent, then $P(E \mid F) = P(E)$.

If events *E* and *F* are dependent then $\boxed{P(E \mid F) = \frac{P(E \cap F)}{P(F)}}$. ☆

This means $P(E \cap F) =$ $P(E \mid F) \cdot P(F)$

If E + F are Indep. $\Rightarrow P(E \cap F) = P(E) \cdot P(F)$

or $P(E) = P(E \mid F)$

Examples:

6. A clothing store that targets young customers (ages 18 through 22) wishes to determine whether the size of the purchase is related to the method of payment. A sample of 300 customers was analyzed and the information is below:

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

a. If a customer is selected at random from this group of customers, what is the probability that the customer paid cash?

$$\frac{100}{300} = \frac{1}{3}$$

b. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card?

$$\frac{130}{300} = \frac{13}{30}$$

c. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan? $\frac{70}{300} = \frac{7}{30}$

d. If a customer is selected at random from this group of customers, what is the probability that the customer purchased under \$40? $\frac{1}{3}$

e. If a customer is selected at random from this group of customers, what is the probability that the customer purchased \$40 or more? $\frac{2}{3}$

f. If a customer is selected at random from this group of customers, what is the probability that the customer paid with a credit card given that the purchase was under $40?

$$P(cc \mid \text{under } 40) = \frac{30}{100} = \frac{3}{10}$$

$$\frac{P(cc \cap \text{under})}{P(\text{under}40)} = \frac{30/300}{1/3} = \frac{3}{10}$$

g. If a customer is selected at random from this group of customers, what is the probability that the customer paid with the layaway plan given that the purchase was $40 or more?

$$P(L \mid \text{over } 40) = \frac{60}{200} = 3/10$$

7. Determine if events A and B are independent.

$P(A)\cdot P(B) = P(A\cap B)$

a. $P(A)=0.9$, $P(B)=0.3$, $P(A\cap B)=0.27$

do these

b. $P(A)=0.4$, $P(B)=0.6$, $P(A\cap B)=0.20$

Popper 01

Let $P(E)=0.44, P(F)=0.47, P(E\cup F)=0.72$

9. $P(E\cap F)=$

$P(E\cup F) = P(E) + P(F) - P(E\cap F)$

$.72 = .44 + .47 - X$

a. 0.43
b. 0.40
c. 0.19
d. 0.75
e. none of these

10. $P(E\mid F)=$ $\dfrac{P(E\cap F)}{P(F)} = \dfrac{.19}{.47}$

a. 0.43
b. 0.40
c. 0.19
d. 0.75
e. none of these