Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

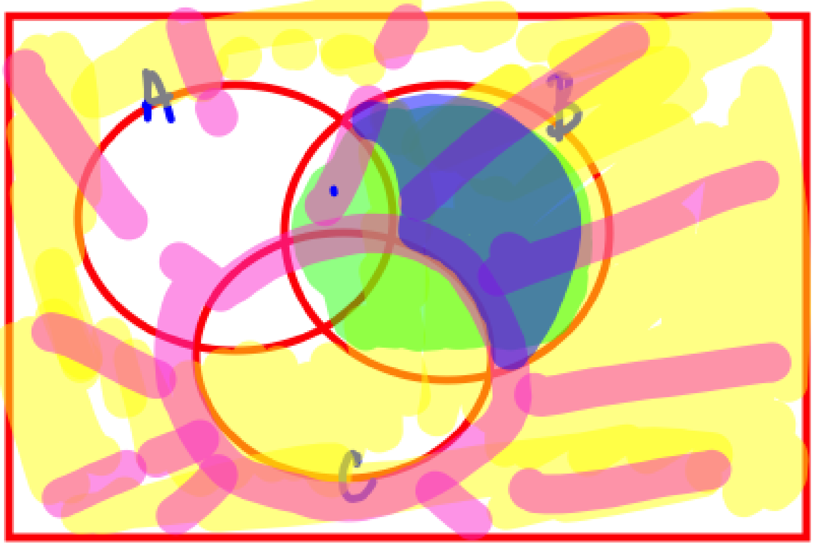
And by appointment

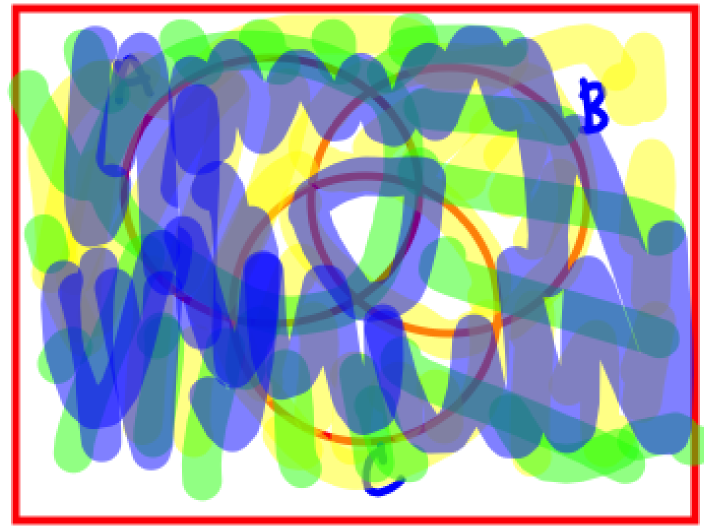
Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

$A^c \cap B \cap C^c$

$\Rightarrow (B \cap C)^c \cup A^c$

$\langle B^c \cup C^c \cup A^c \rangle$





$(B \cap C^c)^c$

$= B^c \cup C$

Math 2311
Sections 3.1 – 3.2

Popper 03

$K + 3K + 2K + K + 2K = 1$

$9K = 1$

$K = 1/9$

Given the following sampling distribution:

X	0	1	2	3	4
P(X)	k	3k	2k	k	2k
	1/9	1/3	2/9	1/9	2/9

1. What is the value of *k*?
   a. 9/10 b. 1/9 c. 9 d. none of these

$P(X=3)$ or $P(X=4)$

2. What is P(X>2)?
   a. 1/3 b. 4/9 c. 5/9 d. none of these

$.15 + .05 + .1 + x + .1 + .15 + .15$
$= 1$

Example: Suppose you are given the following distribution table:

X^2	1	4	9	16	25	36	49
X	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(*X*)	**0.15**	**0.05**	**0.10**	**?**	**0.10**	**0.15**	**0.15**

$= .3$

Find the following:

P(X = 4) $= .3$

P(X < 2) $.15$

P(2 < X ≤ 5) $.5$

P(X > 3) $.7$ or $= 1 - P(X \le 3)$
$1 - .3 = .7$

$P(X \le 2) = .2$

$\star$ $P(X < c) = 1 - P(X \ge c)$
$P(X > c) = 1 - P(X \le c)$
$P(X \le c) = 1 - P(X > c)$
$P(X \ge c) = 1 - P(X < c)$

What is the expected value?

$E[X] = 1(.15) + 2(.05) + \cdots + 7(.15) = 4.2$

The variance and standard deviation?

$Var[X] = E[X^2] - (E[X])^2$
$21.3 - (4.2)^2 = 3.66$

$S = \sqrt{3.66} = 1.91$

Rules for means and variances:

Suppose X is a random variable and we define W as a new random variable such that $W = aX + b$, where a and b are real numbers. We can find the mean and variance of W with the following formula:

ex: $E[3X+5]$

$3 \cdot E[x] + 5$

$$E[W] = E[aX+b] = aE[X]+b$$

$$\sigma_W^2 = Var[W] = Var[aX+b] = a^2 Var[X]$$

$(sd \cdot a)$

Likewise, we have a formula for random variables that are combinations of two or more other independent random variables. Let X and Y be independent random variables,

$$E[X+Y] = E[X]+E[Y]$$

$$\sigma_{X+Y}^2 = Var[X+Y] = Var[X]+Var[Y]$$

and

$$E[X-Y] = E[X-Y] = E[X]-E[Y]$$

$$\sigma_{X-Y}^2 = Var[X-Y] = Var[X]+Var[Y]$$

$E[X] = 22 \quad S_x = 3$

#14 from text: Suppose you have a distribution, X, with mean = 22 and standard deviation = 3. Define a new random variable $Y = 3X + 1$.

$Var[X] = 9$

a. Find the variance of X. 9

b. Find the mean of Y.

$$E[Y] = E[3X+1] = 3E[X] + 1$$
$$= 3(22) + 1 = 67$$

c. Find the variance of Y.

$$Var[Y] = Var[3X+1] = 9Var[X]$$
$$= 9 \cdot 9 = 81$$

d. Find the standard deviation of Y.

$$= \sqrt{81} = 9$$

From Quiz: 3

Suppose you want to play a carnival game that costs 8 dollars each time you play. If you win, you get $100. The probability of winning is 3/100. What is the expected value of the amount that you, the player, stand to gain?

	win	lose
X	$92	−$8
$P(x)$	$\frac{3}{100}$	$\frac{97}{100}$

$$E[X] = 92\left(\frac{3}{100}\right) + (-8)\left(\frac{97}{100}\right)$$

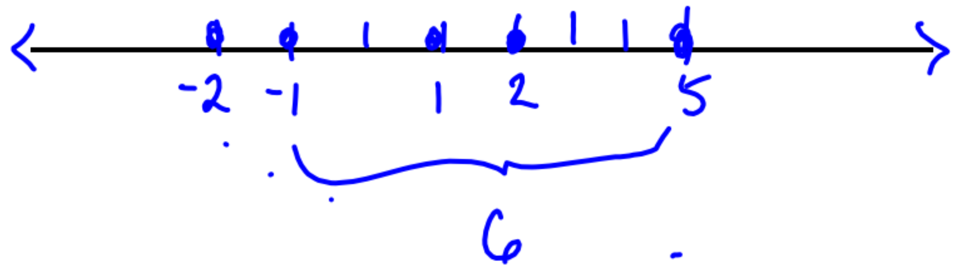
$$= -\$5$$

−1, 1 same 1, −1 order doesn't matter

A random sample of 2 measurements is taken from the following population of values: -2, -1, 1, 2, 5. What is the probability that the range of the sample is 6?

(−2, −1) or (−2, 1) or (−2, 2)

$_5C_2 = 10$ $\boxed{1/10}$



range = max − min

Popper 03

Given the following sampling distribution:

X	-16	-13	-7	11	14
P(X)	$^2/_{25}$	$^1/_{25}$	$^2/_{25}$	$^7/_{100}$	$\boxed{\_\_}$

$8/100 \quad 4/100 \quad 8/100 \quad 7/100$

$1 - \frac{27}{100}$

$1 - .27$

3. P(X = 14) =
   a. 0.73 b. 0.08 c. 0.27 d. none of these

4. What is the mean of this sampling distribution?
   a. 1.67 b. -1.59 c. 8.63 d. none of these

$E[X] = -16(2/25) - 13(1/25) - 7(2/25) + 11(7/100) + 14(\quad)$

5. Suppose Y = 3x + 2. What is the mean of Y?
   a. 25.89 b. 27.89 c. 8.63 d. none of these

$E[Y] = 3E[X] + 2$

Section 3.2:

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
2. The probability of success is p and probability of failure is $q = 1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable.**

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

The random variable X = number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials. The possible values of X are whole numbers that range from 0 tc n. As an abbreviation, we say $X \sim B(n, p)$.

n = # of trials
p = prob. of success

= d

≤ p

Binomial probabilities are calculated with the following formula:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}={}_nC_k\,p^k(1-p)^{n-k}$$

In R, $P(X=k)=$ dbinom(k,n, p).

With a TI-83/84 calculator, $P(X=k)=$ binompdf(n, p, k)

Example: A fair coin is flipped 30 times. $n=30$

What is the probability that the coin comes up heads exactly 12 times?

$P(X=12)=$ dbinom(12,30,½) $p=1/2$

binompdf(30,½,12) = .08055

$P(X\le k)=$ pbinom(k,n,p) $P(X\le k)=$ binomcdf(n, p, k)

What is the probability the coin comes up heads less than 12 times?

$P(X<12)=P(X\le 11)$

binomcdf(30, ½, 11)

0 1 2 3 4 5 . . . . 11 12 | 13 . . .

$\underbrace{0\ 1\ 2\ \cdots\ 12}_{\text{compliment}}$

What is the probability the coin comes up heads more than 12 times?

$P(X>12) = 1 - P(X \le 12)$

$= 1 - \text{pbinom}(12, 30, 1/2)$

$1 - \text{binomcdf}(30, 1/2, 12)$

$= \boxed{.8192}$

The mean and variance of a binomial distribution are computed using the following formulas:

$\mu = E[X] = np$ mean

$\sigma^2 = np(1-p)$ variance

for above problem

$\mu = E[X] = 30(1/2) = 15$

$\sigma^2 = 30(1/2)(1/2)$

$= 15/2$

$$P(X < c) = P(X \le c-1)$$

$$P(X > c) = 1 - P(X \le c)$$

$$P(X \ge c) = 1 - P(X \le c-1)$$

need $\le$ to use commands

$$\overbrace{1 \quad 2 \quad 3}\ \overbrace{4 \quad 5 \quad 6}$$

$$P(X < 4) = P(X \le 3)$$

$p = .8 \quad n = 5$

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

a. No one will contract the flu?

$P(X=0) = \text{binompdf}(5, .8, 0)$

$\text{dbinom}(0, 5, .8)$

$= .00032$

b. All will contract the flu?

$P(X=5)$

c. Exactly two will get the flu?

$P(X=2)$

0 1 2 3 4 5

d. At least two will get the flu?

$$P(X \geq 2) = 1 - P(X \leq 1)$$

e. Let X = number of family members contracting the flu. Create the probability distribution table of X.

X	0	1	2	3	4	5
$p(x)$						

f. Find the mean and variance of this distribution.

$$E[X] = 5(.8)$$

$$\sigma^2 = 5(.8)(.2)$$

Popper 03

$\nearrow .06 \quad n = 11$

6. In testing a new drug, researchers found that 6% of all patients using it will have a mild side effect. A random sample of 11 patients using the drug is selected. Find the probability that none will have this mild side effect.

$P(X = 0) = \text{binompdf}(11, .06, 0)$

$\text{dbinom}(0, 11, .06)$

a. 0.0609 b. 0.5063 c. 0.4937 d. 0.3063

7. Suppose you have a binomial distribution with *n* = 20 and *p* = 0.4. Find P(8 ≤ X ≤ 12).

a. 0.3834 b. 0.5631 c. 0.9790 d. 0.5955