Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

Recall:

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

The random variable X = number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials. The possible values of X are whole numbers that range from 0 tc n. As an abbreviation, we say $X \sim B(n, p)$.

Section 3.3:

The **geometric distribution** is the distribution produced by the random variable *X* definec to count the number of trials needed to obtain the first success.

For example: Flipping a coin until you get a head
Rolling a die until you get a 5

A random variable *X* is geometric if the following conditions are met:

1. Each observation falls into one of just two categories, “success” or “failure.”
2. The probability of success is the same for each observation.
3. The observations are all independent.
4. The variable of interest is the number of trials required to obtain the first success.

Notice that this is different from the binomial distribution in that the number of trials is unknown. With geometric distributions we are trying to determine how many trials are needed in order to obtain a success.

The probability that the first success occurs on the n^{th} trial is

$$P(X=n)=(1-p)^{n-1}p$$

fail fail fail success

where p is the probability of success.

The probability that it takes *more* than n trials to see the first success is

$P(X>n)=(1-p)^n$

$P(X \le n) = \text{pgeom}(n-1, p)$

R commands: $P(X=n)=$ dgeom(n – 1, p) and $P(X>n)=$1-pgeom(n – 1, p)

$P(X \le n) = \text{geometcdf}(p, n)$

Ti-83/84 commands: $P(X=n)=$ geometpdf(p, n) and $P(X>n)=$1- geometcdf (p, n)

The mean, or expected number of trials to get a success in a geometric distribution is

$E[X]=\mu=\dfrac{1}{p}$ and the variance is $\sigma^2=\dfrac{1-p}{p^2}$.

$$\sigma = \frac{\sqrt{1-p}}{p}$$

Examples: $p = .44 \quad 1-p = .56$

From text: **#8.** A quarter back completes 44% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass.

a. What is the probability that the quarterback throws 3 incomplete passes before he has a completion?

$$(.56)(.56)(.56)(.44) = .0773 \quad P(X=4)$$

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geometpdf(.44,4)
                    .07727104
                    .07727104
```

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> dgeom(3,.44)
[1] 0.07727104
```

b. How many passes can the quarterback expect to throw before he completes a pass?

$$E[X] = \frac{1}{p} = \frac{1}{.44} = 2.27$$

c. Determine the probability that it takes more than 5 attempts before he completes a pass.

$$P(X>5) = (1-.44)^5 = .0551$$

d. What is the probability that he attempts more than 7 passes before he completes one?

$$P(X>7) = (1-.44)^7 = .0173$$

$p = .6$

#14. Newsweek in 1989 reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming a random sample from the population of all school children at risk, find:

a. The probability that at least 5 children out of 10 in a sample taken from a school may have a blood lead level that may impair development. Binomial $n = 10$

$$P(X \geq 5) = 1 - P(X \leq 4) = 1 - \text{pbinom}(4, 10, .6)$$

$0\ 1\ 2\ 3\ 4\ \underbrace{5\ 6 \cdots 10}$

$$1 - \text{binomcdf}(10, .6, 4)$$

$$= .8338$$

b. The probability you will need to test 10 children before finding a child with a blood lead level that may impair development. Geometric

$$P(X = 11) = \text{dgeom}(10, .6)$$

$$\text{geometpdf}(.6, 11)$$

6.29×10^{-5}

$.00006$

c. The probability you will need to test no more than 10 children before finding a child with a blood lead level that may impair development. Geometric

$$P(X \leq 10) = \text{geometcdf}(.6, 10)$$

$$\text{pgeom}(9, .6)$$

$.9999$

Popper 04

1. Which of the following is true?
   a. Binomial has a set number of trials
   b. Binomial has only success or fail
   c. Geometric has only success or fail
   d. Geometric does not have a set number of trials (we are looking for first success)
   e. All of the above are true

2. Joe has an 50% probability of passing his statistics quiz 4 each time he takes it. What is the probability he will take no more than 5 tries to pass it? Geomt.
   a. 0.9844
   b. 0.0156
   c. 0.6289
   d. 0.0034
   e. none of these

$P(X \leq 5) = .96875$

geometcdf (.5, 5)

pgeom (4, .5)

Review (Quiz 4 material)

$\to \text{var} = 9$

Suppose you have a distribution, X, with mean = 10 and standard deviation = 3. Define a new random variable Y = 5X - 5. Find the mean and standard deviation of Y.

$$E[Y] = 5E[X] - 5 = 5(10) - 5 = 45$$

$$VAR[Y] = 5^2 \, Var[X] = 25 \cdot 9$$

$$S_y = \sqrt{25 \cdot 9} = 15 \quad (5 \cdot sd_x = 5 \cdot 3 = 15)$$

In testing a new drug, researchers found that 6% of all patients using it will have a mild side effect. A random sample of 11 patients using the drug is selected. Find the probability that none will have this mild side effect.

$n = 11 \quad p = .06 \quad P(X=0) = \text{binompdf}$

$\text{binompdf}(11, .06, \underline{\underline{0}}) = .506$

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dbinom(0,11,.06)
[1] 0.5062982
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Find the probability that at least one will have this mild side effect. $P(X \geq 1) = 1 - P(X=0)$

Find the probability that exactly two will have this mild side effect

$P(X=2)$

Suppose you have a binomial distribution with n = 41 and p = 0.4. Find $P(8 \le X \le 12)$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

$P(X \le 12) - P(X \le 7)$

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pbinom(12,41,.4)-pbinom(7,41,.4)
[1] 0.1040156
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binomcdf(41,.4,12)-binomc▸
.1040156076
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Each year a company selects a number of employees for a management training program. On average, 40 percent of those sent complete the program. Out of the 29 people sent, what is the probability that 7 or more complete the program?

n

Binomial

$$P(X \ge 7) = 1 - P(X \le 6)$$

$$1 - \text{binomcdf}(29, .4, 6)$$

A quarter back completes 62% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass. What is the probability that the quarterback throws 4 incomplete passes before he has a completion?

Geometric

$p = .62$ $P(X=5)$

Joe has an 14% probability of passing his statistics quiz 4 each time he takes it. How many times should Joe expect to take his quiz before passing it?

Geometric

$p = .14$ $E[X] = \frac{1}{p}$

mean

Popper 04

Let $P(A)=0.44, P(B)=0.47, P(A\cup B)=0.79$

3. $P(A\cap B)=$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

a. 0.160
b. 0.255
c. 0.120
d. 0.273
e. none of these

4. $P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$

a. 0.160
b. 0.255
c. 0.120
d. 0.273
e. none of these

5. $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$

a. 0.160
b. 0.255
c. 0.120
d. 0.273
e. none of these