

Math 2311

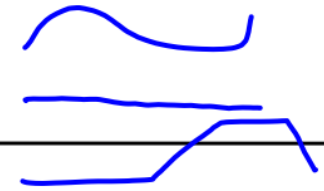
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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: <http://www.math.uh.edu/~bekki/Math2311.html>

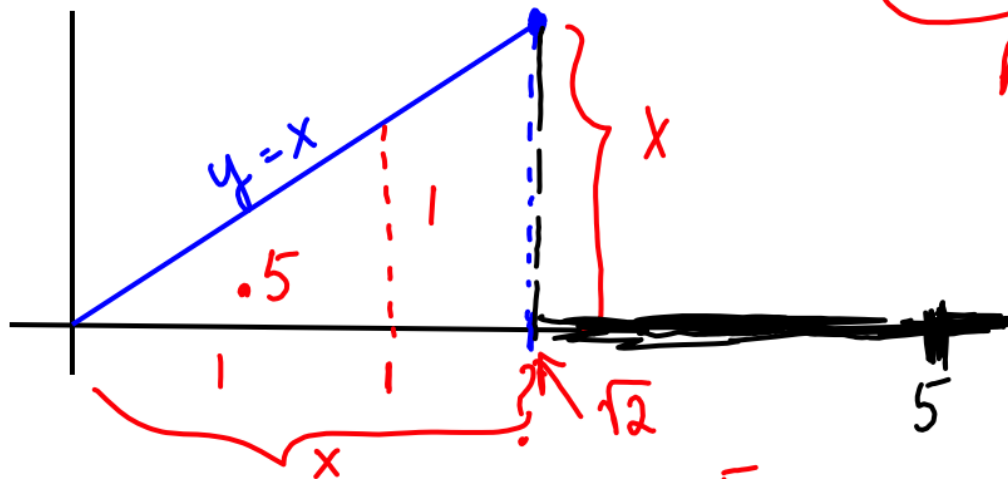


4.1 - Density Curves

What do we know about density curves? *area under curve = 1*

Example: Suppose we have a density curve defined for $x \geq 0$ defined by the line $y = x$.

Sketch:



$$A = \frac{1}{2} x \cdot x = \frac{1}{2} x^2$$

$$1 = \frac{1}{2} x^2$$

$$2 = x^2$$

$$x = \sqrt{2}$$

What percent of observations fall below 1? *.5*

What is the probability that $X \geq 5$? *0*

Where is the median of this curve?

$$A = .5$$

$$\frac{1}{2} x^2 = .5$$

$$x^2 = 1$$

$$x = 1$$

4.2 - The Normal Distribution

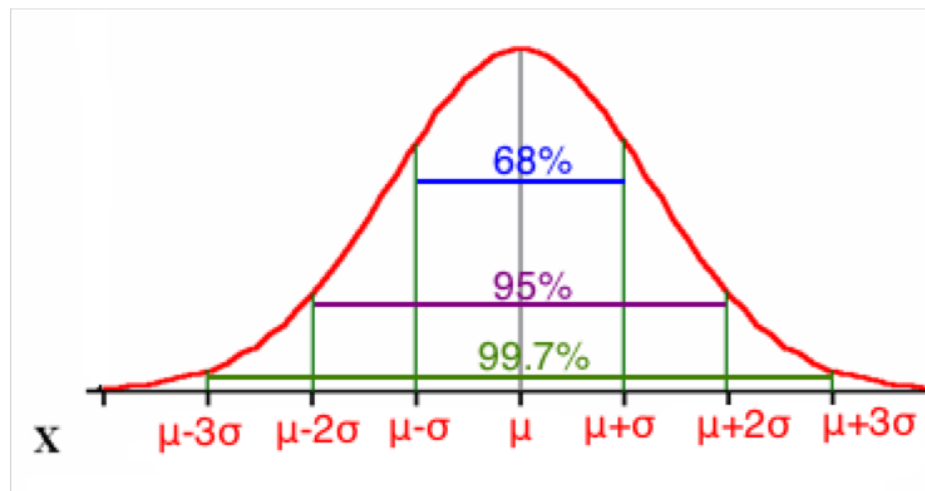
A density curve that is symmetric, single peaked and bell shaped is called a **normal distribution**.

The normal distribution with mean μ and standard deviation σ is represented by $N(\mu, \sigma)$.

The Empirical Rule:

The Empirical Rule states if a distribution has a normal distribution,

1. Approximately 68% of all observations fall within one standard deviation of the mean.
2. Approximately 95% of all observations fall within two standard deviations of the mean.
3. Approximately 99.7% of all observations fall within three standard deviations of the mean.

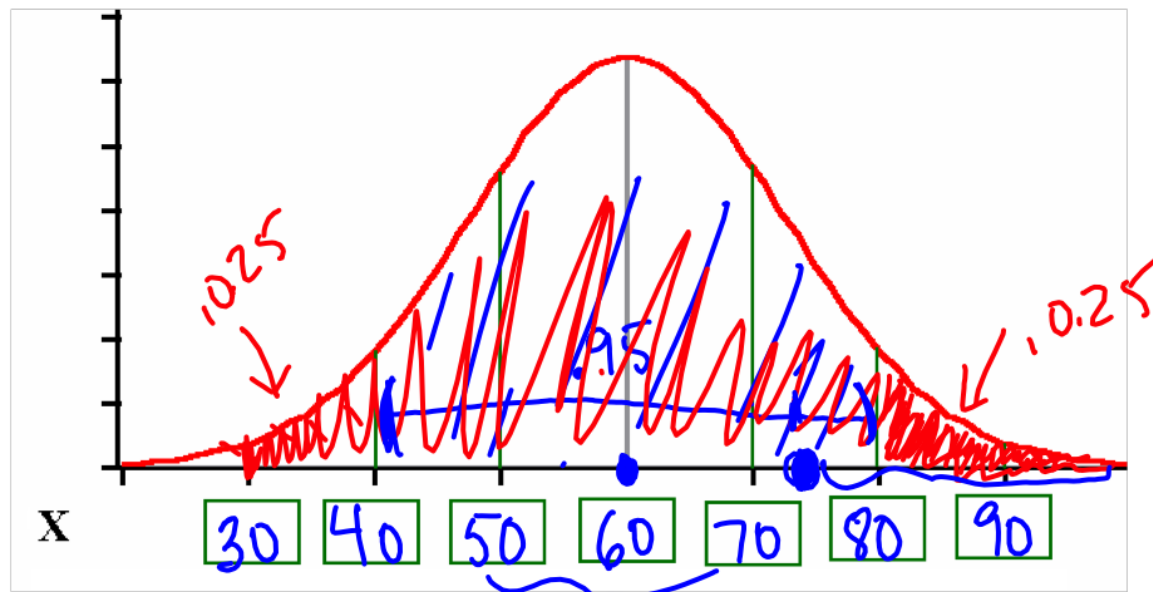


Example:

The length of time needed to complete a certain test is normally distributed with mean 60 minutes and standard deviation 10 minutes.

What is the probability that someone will take between 40 and 80 minutes to complete the test? Sketch the distribution and shade in the area in question.

.95



Find the interval that contains the middle 68% of completion times for all people taking the test.

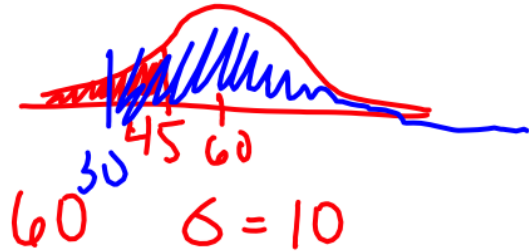
[50, 70]

What percent of people take more than 80 minutes to complete the test?

.025

What if our values are not exactly within one, two or three standard deviations from the mean? Probabilities for these can still be found a number of ways, one of which we will explore in the next section. Using R, the probability can be found with the command `pnorm(X, μ , σ)`. Note that the command in R only gives the probability that X is less than a given value. If we need to find the probability that X is greater than the given value, we will need to subtract the answer from 1.

With the TI-83 and TI-84 calculator, the command is `normalcdf(lower_limit, upper_limit, μ , σ)`.



Continuing the example above,

What is the probability that someone will take less than 45 minutes to complete the test?

.0668

$$P(X < 45) = \text{normalcdf}(-100000000, 45, 60, 10) = \text{pnorm}(45, 60, 10)$$

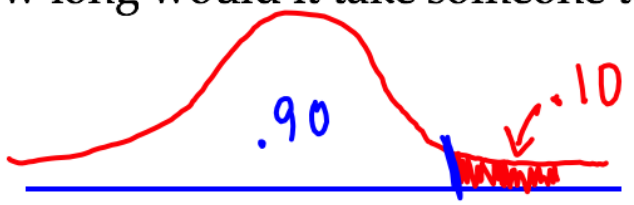
What is the probability that someone will take more than 30 minutes to complete the test?

.999

$$P(X > 30) = \text{normalcdf}(30, 100000000, 60, 10)$$

$$1 - P(X \leq 30) = 1 - \text{pnorm}(30, 60, 10)$$

How long would it take someone to finish the test if they are in the top 10% of the times?



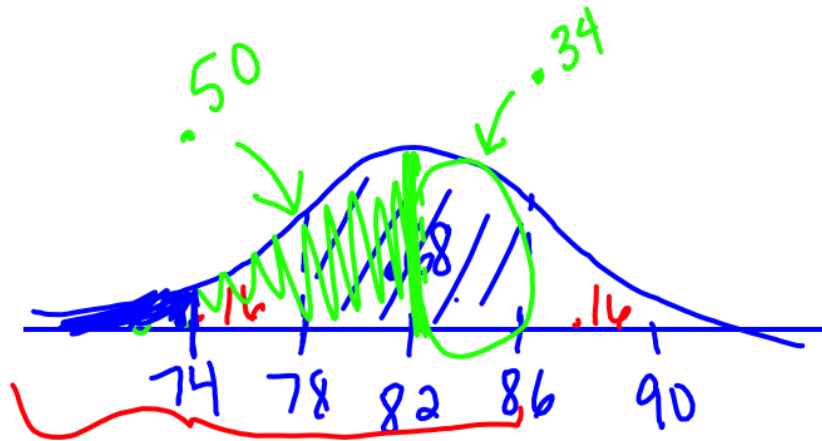
72.8

```
> qnorm(.9,60,10)
[1] 72.81552
```

Popper 07

1. If a group of students have test scores that are normally distributed with a mean of 82 and a standard deviation of 4, half of the students made below a grade of:

- a. 78
- b. 74
- c. 82
- d. 70
- e. none of these



2. If a student made a grade of 86, what is their percentile rank?

- ~~a.~~ 68th percentile
- b. 84th percentile
- c. 90th percentile
- ~~d.~~ 52nd percentile
- ~~e.~~ none of these

3. Find the probability that someone made less than a grade of 74.

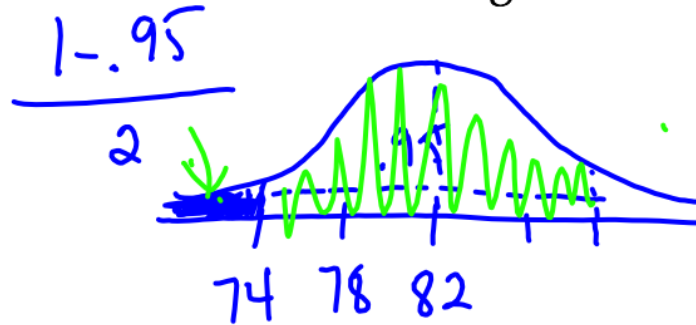
a. 5%

b. 2.5%

~~c. 95%~~

d. 23%

~~e. none of these~~



4.3 - Standard Normal Calculations

As suggested in the previous section, all normal distributions share many common properties. In fact, if change the units to σ and center the graph at $\mu=0$, all normal distributions would be exactly the same. This is called standardizing. If x is an observation from a normal distribution with mean μ and standard deviation σ , the standardized value of x is called the z-score and is computed with the formula below.

$$z = \frac{x - \mu}{\sigma}$$

Z-Score:



A z-score tells us how many standard deviations the observed value falls from the mean.

We can use z-scores to “standardize” values that are on different scales to compare them.

Example: Jill scores 710 on the mathematics part of the SAT. The distribution of SAT scores in a reference population is $N(500, 100)$. Jack takes the ACT mathematics test and scores 29. ACT scores follow the distribution $N(18, 6)$. Find the standardized scores (z-scores) for each. Who do you think performed better on their test?

$$\text{Jill: } x = 710 \quad \mu = 500 \quad \sigma = 100 \quad z = \frac{710 - 500}{100} = 2.1$$

$$\text{Jack: } x = 29 \quad \mu = 18 \quad \sigma = 6 \quad z = \frac{29 - 18}{6} = 1.8\bar{3}$$

When we “standardize” values, they fit the standard normal distribution.

The standard normal distribution is the normal distribution with $N(0,1)$:

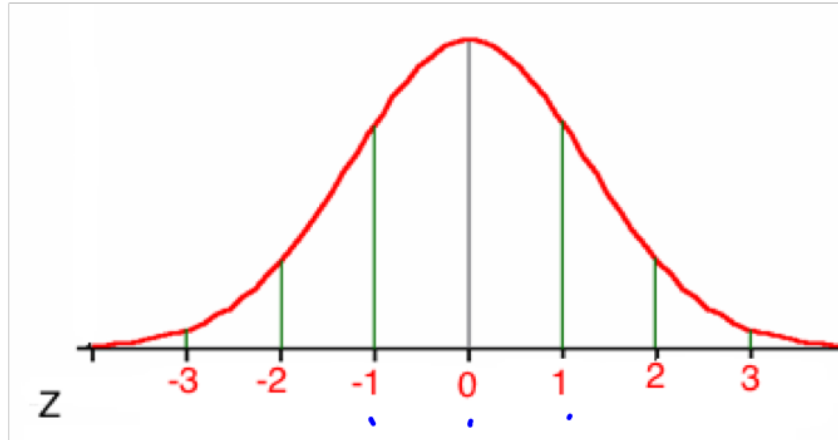


Table A in your appendix gives areas under the standard normal curve for values of z . The table entry for each value of z gives the area under the curve to the left of z – in other words, it gives $p(Z < z)$.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



Example: Using Table A, find the following probabilities:

A. $p(Z < \underline{-1.34}) = .0901$

B. $p(Z < 2.02) = .9783$

C. $p(Z > 1.35) = 1 - P(Z < 1.35) = 1 - .9115 = .0885$

D. $p(-1.34 < Z < 2.02) = P(Z < 2.02) - P(Z < -1.34)$
 $.9783 - .0901 = .8882$

Now let's repeat with calculator and R-Studio.

$$\mu = 0, \sigma = 1$$

If we want to use the table for probabilities and are not given z , we must compute the z -score using the formula above.

Example: If X has distribution $N(90,12)$, standardize X and use Table A to find the following probabilities:

A. $p(X < 80) = P\left(z < \frac{80-90}{12}\right) = P(z < -.83) = .203$

B. $p(X > 95)$ normalcdf(95, 100000, 90, 12)
 $\cdot 338$

```
> pnorm(-.83)
[1] 0.2032694
> pnorm(80,90,12)
[1] 0.2023284
```

C. $p(80 < X < 95)$

$= .459$ $P(x < 95) - P(x < 80)$

normalcdf(-100000, -.83)

0,1

normalcdf(-100000, 80, 90, 12)

Using the calculator and R, it's a bit easier

R-Studio: $P(X < x) = \text{pnorm}(x, \text{mean}, \text{sd})$

Ti-83/84: $\text{normalcdf}(\text{low}, \text{high}, \text{mean}, \text{sd})$

$\rightarrow P(x > \#) = 1 - \text{pnorm}(\#, \text{mean}, \text{sd})$

More Examples:

The length of time needed to complete a certain test is normally distributed with mean 24 minutes and standard deviation 11 minutes. Find the probability that it will take less than 40 minutes to complete the test.

$$\mu = 24 \quad \sigma = 11$$

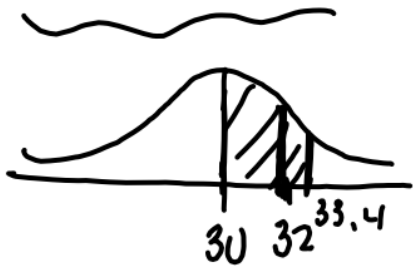
$$P(X < 40)$$

$$\text{normalcdf}(-100000, 40, 24, 11)$$

$$p\text{norm}(40, 24, 11)$$

$$= .927$$

If X is normally distributed with a mean of 30 and a standard deviation of 2, find $P(30 \leq X \leq 33.4)$.



$$\begin{aligned} & \text{normalcdf}(30, 33.4, 30, 2) \\ & \text{pnorm}(33.4, 30, 2) - \text{pnorm}(30, 30, 2) \\ & = .455 \end{aligned}$$

Suppose that X is normally distributed with a mean of 60 and a standard deviation of 9. What is $P(X \geq 68.73)$?

↑

$$\begin{aligned} & \text{normalcdf}(68.73, 100000, 60, 9) \\ & 1 - \text{pnorm}(68.73, 60, 9) \end{aligned}$$

```
> 1-pnorm(68.73,60,9)
[1] 0.1660232
```

Suppose that x is normally distributed with a mean of 20 and a standard deviation of 3.
What is $P(16.91 \leq x \leq 24.59)$?

$$\begin{aligned} & \text{normalcdf}(16.91, 24.59, 20, 3) \\ & \text{pnorm}(24.59, 20, 3) - \text{pnorm}(16.91, 20, 3) \\ & .785 \end{aligned}$$

Find a value of c so that $P(Z \leq c) = 0.47$.

```
invNorm(.47,0,1)
-.0752698548
```

```
> qnorm(.47)
[1] -0.07526986
```

Popper 07

4. A normal density curve has which of the following properties?
- It is symmetric.
 - It has a peak centered above its mean.
 - The spread of the curve is proportional to its standard deviation.
 - All of the properties above are correct.
 - None of the properties above are correct.

5. The area under the standard normal curve corresponding to $-0.3 < Z < 1.6$ is

- 0.3273
- 0.4713
- 0.5631
- 0.9542

Same as asking
 $P(-.3 < Z < 1.6)$