

Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: <http://www.math.uh.edu/~bekki/Math2311.html>

Last week we said a **confidence interval** is a range of possible values that is likely to contain the unknown population parameter that we are seeking.

And the formula for a confidence interval is: sample statistic \pm margin of error

Confidence Interval for a Proportion $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

↑ sample proportion

Conditions:

- 1. The sample must be an SRS from the population of interest.
- 2. The population must be at least 10 times the size of the sample.
- 3. The number of successes and the number of failures must each be at least 10 (both $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$).

7.3 - Confidence Interval for the Difference of Two Proportions

The assumptions that need to be satisfied for a two-sample proportion are slightly different than those for a one-sample.

1. Both samples must be independent SRSs from the populations of interest.
2. The population sizes are both at least ten times the sizes of the samples.
3. The number of successes and failures in both samples must all be ≥ 10 .

To make the comparison, we will need to find the difference of the two proportions, $\hat{p}_1 - \hat{p}_2$. The

standard error for this difference is $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$. So our formula for the confidence interval is:

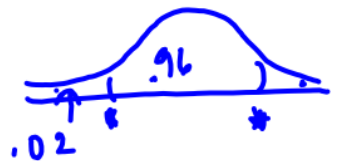
$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

\uparrow
invNorm $\left(\frac{1+CL}{2} \right)$
g norm

Example:

The National Research Council of the Philippines reported that 210 of 361 members in biology are women, but only 34 of 86 members in mathematics are women. Establish a 96% confidence interval estimate of the difference in proportions of women in biology and mathematics in the Philippines. Interpret your results.

$$\begin{aligned} 1: \text{biology} \quad n_1 &= 361 & \hat{p}_1 &= \frac{210}{361} \\ 2: \text{math} \quad n_2 &= 86 & \hat{p}_2 &= \frac{34}{86} \end{aligned}$$

96% CI 

$$z^* = \text{invNorm}\left(\frac{1.96}{2}\right)$$
$$z^* = 2.054$$

$$n_1 \hat{p}_1 = 361 \left(\frac{210}{361}\right) = 210 \geq 10$$

$$n_1 (1 - \hat{p}_1) = 361 \left(\frac{151}{361}\right) = 151 \geq 10$$

$$n_2 \hat{p}_2 = 86 \left(\frac{34}{86}\right) = 34 \geq 10$$

$$n_2 (1 - \hat{p}_2) = 86 \left(\frac{52}{86}\right) = 52 \geq 10$$

$$\left(\hat{p}_1 - \hat{p}_2\right) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

margin of error (ME)

$$ME = (2.054)(.0588) = \underline{\underline{.1208}}$$

Stand. error:

$$\sqrt{\frac{\frac{210}{361} \left(1 - \frac{210}{361}\right)}{361} + \frac{\frac{34}{86} \left(1 - \frac{34}{86}\right)}{86}}$$
$$= \underline{\underline{.0588}}$$

$$\left(\frac{210}{361} - \frac{34}{80}\right) \pm \text{ME}$$

$$.1864 \pm .1208$$

$$[.0656, .3072]$$

We are 96% confident that true difference in proportion of women in biology and women in math in the Philippines is between .0656 and .3072

7.4 - Confidence Interval for a Population Mean

population μ
Sample statistic \bar{x}

Recall that formula for a confidence interval is $\text{statistic} \pm \text{margin of error}$. When we are making an inference about a population mean, the statistic will be our sample mean, \bar{x} .

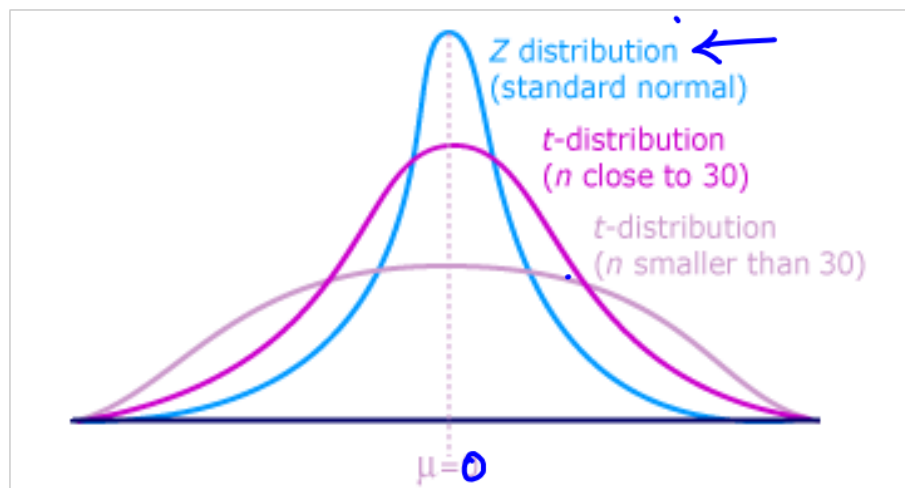
The critical value we use to find the margin of error for our calculation will be based on whether the population or sample standard deviation is known. When the population standard deviation is

known, we use the formula $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$ and when it is unknown, we will need to find the sample

standard deviation, s , and use the formula $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ where t^* is the t -critical value based on $n - 1$ degrees of freedom.

So, what is t^* ?

t -distribution vs. standard normal distribution:



degrees of freedom
n-1

How do we find critical values for a t -distribution?

$$\text{invT} \left(\frac{1+CL}{2}, df \right)$$

~~qt~~

Commands:

TI-83 – there is no invT command!!

TI-84 – invT under DIST. Enter lower tail and degrees of freedom

Example: for a 90% CI with 10 degrees of freedom, we will use invT(.95,10)

R-Studio – the command is qt

Example: for a 90% CI with 10 degrees of freedom, we will use qt(.95,10) ←

Appendix has a t distribution table too.

The assumptions for a population mean are:

1. The sample must be an SRS from the population of interest.
2. The data must come from a normally distributed population. If this is not the case or if we are unsure whether the population is normally distributed, the sampling distribution of \bar{x} must be normally distributed. (Recall from section 4.4 that we can assume that the sampling distribution of \bar{x} is normal for values of n greater than 30.)

z^*

Examples:

1. Suppose your class is investigating the weights of Snickers 1-ounce fun-size candy bars to see if customers are getting full value for their money. Assume that the weights are normally distributed with standard deviation $\sigma = .005$ ounces. Several candy bars are randomly selected and weighed with sensitive balances borrowed from the physics lab.

The weights are: .95 1.02 .98 .97 1.05 1.01 .98 1.00

We want to determine a 90% confidence interval for the true mean, μ .

← add up + div. by 8
 $n=8$

a. What is the sample mean? $\bar{x} = .995$

b. Determine z^* . 1.645 $\text{invNorm}\left(\frac{1.90}{2}\right)$ $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

c. Determine the 90% confidence interval. (Show your work) $.995 \pm 1.645 \left(\frac{.005}{\sqrt{8}}\right)$

d. Write a sentence that explains the significance of the confidence interval.

$$.995 \pm .0029$$

$$\underline{\underline{[.9921, .9979]}}$$

We are 90% confident that the true mean weight of funsize Snickers is between .9921 oz and .9979 oz.

$$n=16 \quad \bar{x}=500 \quad s=100 \quad t^*$$

2. A SRS of 16 seniors from HISD had a mean SAT-math score of 500 and a standard deviation of 100. We know that the population of SAT-math scores for seniors in the district is approximately normally distributed.

a. Find the 90% confidence interval for the mean SAT-math score for the population of all seniors in the district.

$$t^* = \text{invT}(.95, 15) = 1.7531$$

Handwritten notes: 1.9/2, n-1 (with arrows pointing to .95 and 15); 9+ (with an arrow pointing to 15)

b. Explain the meaning of the above confidence interval.

$$a. \quad \bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$500 \pm 1.7531 \left(\frac{100}{\sqrt{16}} \right)$$

$$500 \pm 43.825$$

$$[456.175, 543.825]$$

b. We are 90% confident that the true mean SAT math score for seniors in HISD is between 456.175 and 543.825

Interpretation of confidence level
(repeated)
if we took 100 samples of size 16
90 of them ~~are~~ would contain
the true mean.

Popper 17

You have measured the systolic blood pressure of a random sample of 25 employees of a company located near you. A 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138).

1. What is the mean of this sample?
 - a. 122
 - b. 138
 - c. 130
 - d. 129
 - e. none of these

$$130 \pm \underline{8} \leftarrow ME$$

$$130 \pm 2.064 \frac{s}{\sqrt{25}}$$

$$8 = 2.064 \cdot \frac{s}{5}$$

2. If we did not know the population standard deviation but we know the distribution is approximately normal, which critical value will we use for the confidence interval?
 - a. z^*
 - b. t^*
3. What is the value of the test statistic used for the 95% CI?
 - a. 2.064
 - b. 1.96
 - c. 1.645
 - d. 1.753
 - e. none of these

4. What is the sample standard deviation for this example?

- a. 3.88
- b. 19.38**
- c. 21.25
- d. 20.5
- ~~e. none of these~~

$$130 \pm \underline{8}$$

$$\begin{aligned} 8 &= \frac{2x}{5} \\ 5 \cdot 4 &= x/5 \cdot 5 \\ 20 &= x \end{aligned}$$

$$8 = 2.064 \cdot \frac{S}{\sqrt{25}} \quad (\text{solve for } S)$$

5. Would the 90% CI be larger or smaller in width than the 95% CI?

- a. Larger
- b. Smaller

Find the 90% CI

$$\bar{x} = 130$$

$$\text{new } t^* = 1.7109$$

$$S = 19.38$$

$$n = 25$$

$$130 \pm 1.7109 \cdot \frac{19.38}{\sqrt{25}}$$

$$130 \pm 6.6314$$

$$[123.3686, 136.6314]$$