Math 2311

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Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

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1. The p-value of a test of significance is the probability that:

   a) the decision resulting from the test is correct.

   b) 95% of the confidence intervals will contain the parameter of interest.

   c) the null hypothesis is true given the information about the population is true.

   d) the alternative hypothesis is true.

   e) None of these describes the p-value.

probability that the sample data is what it is assuming H_0 is true.

2. If we want to claim that a population parameter is different from a specified value, this situation can be considered as a one-tailed test.

   $\neq$

   a) True

   b) False

8.2 - Inference for a Population Proportion

For these inferences, p_0 represents the given population proportion.

$$H_0 : p = p_0$$

$$H_a : p \neq p_0 \quad or \quad p < p_0 \quad or \quad p > p_0$$

p_0 = claimed population proportion

Conditions:

1. The sample must be an SRS from the population of interest.
2. The population must be at least 10 times the size of the sample.
3. The number of successes and the number of failures must each be at least 10 (both $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$).

$n\hat{p} \geq 10$

Recall, the statistic used for proportions is $\boxed{\hat{p} = \dfrac{\text{\# of successes}}{\text{\# of observations}}}$.

For tests involving proportions that meet the above conditions, we will use the z – test statistic

$$z = \frac{\hat{p} - p_o}{\sqrt{\dfrac{p_o(1-p_o)}{n}}}$$

where

Examples:

3. A new shampoo is being test-marketed. A large number of 16-ounce bottles were mailed out at random to potential customers in the hope that the customers will return an enclosed questionnaire. Out of the 1,000 returned questionnaires, 575 indicated that they like the shampoo and will consider buying it when it becomes available on the market. Perform a hypothesis test to determine if the proportion of potential customers is more than 50%.

$\alpha = .05$

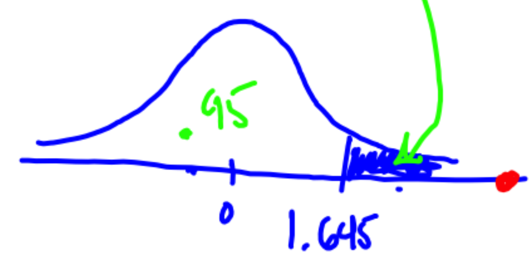
$H_0: p = .5$

$H_a: p > .5$

$\hat{p} = \frac{575}{1000}$

$\hat{p} = .575$

$n = 1000$



$$z = \frac{.575 - .5}{\sqrt{\frac{.5(1-.5)}{1000}}} = 4.743$$

pvalue: $p(z > 4.743) = .00000105 \approx 0 < \alpha$

Overwhelming evidence to reject the null hypothesis and conclude that more than 50% are potential buyer of this shampoo.

7. Mars Inc. claims that they produce M&Ms with the following distributions:

Brown	30%	Red	20%	Yellow	20%
Orange	10%	Green	10%	Blue	10%

A bag of M&Ms was randomly selected from the grocery store shelf, and the color counts were:

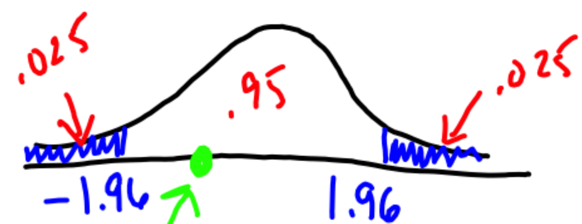
Brown	14	Red	14	Yellow	5
Orange	7	Green	6	Blue	10

$14/56$ $\quad n = 56 \quad \hat{p} = 14/56$

Conduct an appropriate test of the manufacturer's claim for the proportion of brown M&Ms.

$H_0: p = .30$ $\quad \alpha = .05$

$H_a: p \neq .30$



$$z = \frac{14/56 - .3}{\sqrt{\frac{.3(.7)}{56}}} = -.816$$

pvalue: $2p(z < -.816) \doteq .4145 > \alpha$

Fail to reject H_0 which states 30% of M+Ms are brown.

Decision errors can occur when choosing to reject or failing to reject the null hypothesis. There are two types of decision errors; Type I and Type II.
A **Type I error** occurs when you reject the null hypothesis when in fact it is true. This error would correspond to taking action when you would have been better off not doing so. A **Type II error** happens when the null hypothesis is "accepted" (fail to reject) when in fact the alternative hypothesis is true. The Type II error would correspond to taking no action when you would have been better off taking action. The **power** of a test against an alternative hypothesis is the probability that a fixed level, α, significance test will reject the null hypothesis when that particular alternative is true. The following table can help to identify the types of errors.

	H_0 is true	H_a is true
Reject H_0	Type I error	Decision is correct
Fail to reject H_0	Decision is correct	Type II error

Example:

You are considering whether or not to play the lottery with your favorite numbers. What situations denote a type I and II errors?

	H_0 is true (Won't win)	H_a is true (Would win)
Reject H_0 (buy a ticket)	lost \$1 type I	
Fail to reject H_0 (don't buy)		type II

* lump on foot *

3%

Example:

A manufacturer of circuits had observed that, on average, $p = 0.03$ of its circuits failed. One of the engineers suggests changes in the design to try to improve this percentage.

It is decided that $n = 100$ circuits would be made using her method. The company will adopt her method if only zero or one of the circuits failed.

Identify the null and alternative hypotheses at play here. What are the possible Type I and Type II errors the company might make with this strategy?

$H_0: p = .03$ Type I : adopt new method when $p = .03$

$H_a: p < .03$ Type II : fail to adopt new method even though $p < .03$

What is the probability that the company will make a Type I error?

$$P(X=0, p=.03) + P(X=1, p=.03) = \text{binomialcdf}(100, .03, 1)$$

$$P(X \le 1, p = .03) = .195$$

If the actual value for the new method is $p = 0.01$, what is the probability that the company will make a Type II error?

$$P(X>1, p=.01) = 1 - P(X \le 1)$$

$$= 1 - \text{binomialcdf}(100, .01, 1)$$

$$= .264$$

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Suppose that the state said that 65% of all state residents are in favor of a lottery. An SRS of 100 people showed that 52 of those 100 surveyed are in favor of the lottery. Test the claim that the percent in favor of the lottery is less than 6$\overset{5}{0}$%.

3. The null and alternate hypothesis are:
   a. $H_0: p = .65 \quad H_a: p < .65$
   b. $H_0: p = .65 \quad H_a: p > .65$
   c. $H_0: p = .65 \quad H_a: p \neq .65$
   d. $H_0: p = .52 \quad H_a: p < .52$
   e. $H_0: p = .52 \quad H_a: p \neq .52$
   f. none of these

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

4. What is the test statistic?
   a. z= 1.63
   b. z=-1.63
   c. z=-2.72
   d. z=0.98
   e. none of these

5. What is the p-value?
   a.. 0.052
   b. 0.003