Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

Test 3 :

9 questions

7 m/c ← study PT3 Quiz 10-13

2 f/R ← Hyp. Tests

Conf. Intervals & Hyp. Tests

	Ho true	Ho false
Rej	type I	Power
FR		Type II

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1. Rejecting a true null hypothesis is classified as a __________ error.

    a. Type I b. Type II c. Power

2. Failing to reject a false null hypothesis is classified as a _________________ error.

    a. Type I b. Type II c. Power

8.5 – Goodness of Fit Test

χ^2

Suppose we want to make an inference about a group of data (instead of just one or two). Or maybe we want to test counts of categorical data. **Chi-square** (or χ^2) testing allows us to make such inferences.

There are several types of Chi-square tests but in this section we will focus on the goodness-of-fit test. **Goodness-of-fit** test is used to test how well one sample proportions of categories "match-up" with the known population proportions stated in the null hypothesis statement. The Chi-square goodness-of-fit test extends inference on proportions to more than two proportions by enabling us to determine if a particular population distribution has changed from a specified form.

The null and alternative hypotheses do not lend themselves to symbols, so we will define them with words.

H_o: _____ is the same as _____

H_a: _____ is different from _____

For each problem you will make a table with the following headings:

Observed Counts (O)	Expected Counts (E)	$\frac{(O-E)^2}{E}$

The sum of the third column is called the Chi-square test statistic.

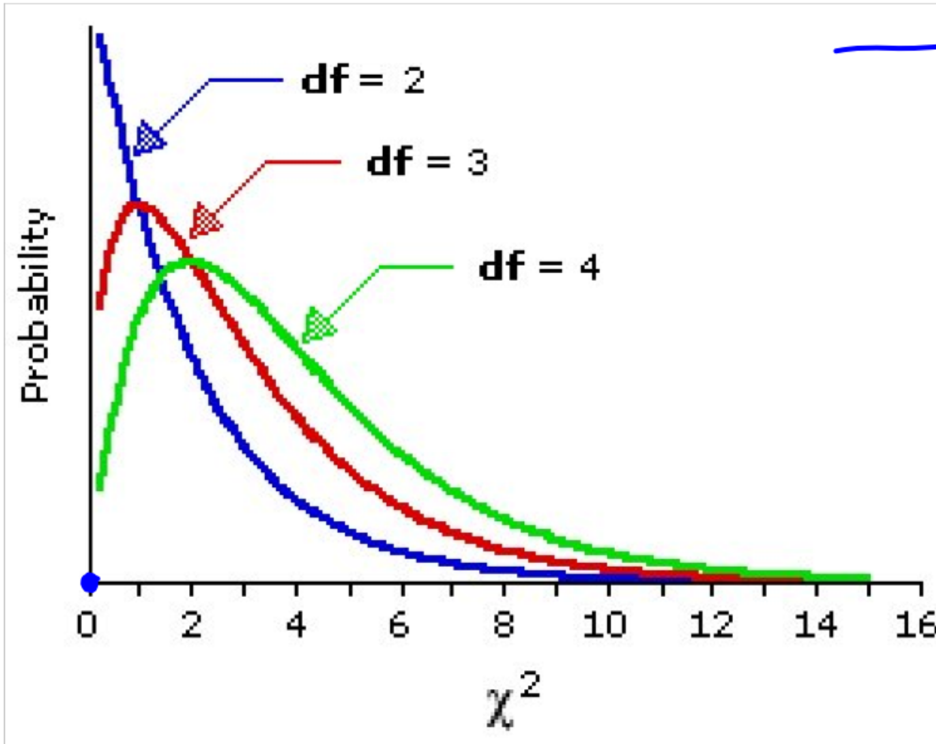
$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

sum

\# of categories

Table D gives *p*-values for χ^2 with $n - 1$ degrees of freedom.

Chi-square distributions have only positive values and are skewed right. As the degrees of freedom increase it becomes more normal. The total area under the χ^2 curve is 1.



χ^2 cdf ← calc

pchisq ← R

The assumptions for a Chi-square goodness-of-fit test are:

1. The sample must be an SRS from the populations of interest.
2. The population size is at least ten times the size of the sample.
3. All expected counts must be at least 5.

To find probabilities for χ^2 distributions:

TI-83/84 calculator uses the command χ^2**cdf** found under the DISTR menu.

R-Studio command is: 1 – pchisq(*test statistic*, *df*)

Examples: $\sum \frac{(O-E)^2}{E} = \chi^2$ $.15(50) = 7.5$

1. The Mixed-Up Nut Company advertises that their nut mix contains (by weight) 40% cashews, 15% Brazil nuts, 20% almonds and only 25% peanuts. The truth-in-advertising investigators took a random sample (of size 50 lbs) of the nut mix and found the distribution to be as follows:

Cashews	*Brazil Nuts*	*Almonds*	*Peanuts*
15 lb	11 lb	13 lb	11 lb

← OBS.

Expected

Cash	Braz	Alm	Pnut
20 lb	7.5	10	12.5

At the 1% level of significance, is the claim made by Mixed-Up Nuts true?

$$\chi^2 = \frac{(15-20)^2}{20} + \frac{(11-7.5)^2}{7.5} + \frac{(13-10)^2}{10} + \frac{(11-12.5)^2}{12.5} = \underline{\underline{3.96}}$$

H_0: the data distribution (of nuts) is same as population

H_a: the data distribution is different from population

pvalue: $p(\chi^2 > 3.96) = \chi^2\text{cdf}(3.96, 999999, 3)$

$1 - \text{pchisq}(3.96, 3)$

$= \underline{.2658} > .01$ Fail to reject Ho

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3. To use the two-sample t procedure to perform a significance test on the difference between two means, we assume
    a. The distributions are exactly normal in each population.
    b. The sample sizes are large. ← $n \geq 30$
    ~~c.~~ The populations' standard deviations are known.
    ✓ d. The samples from each population are independent.
    ~~e. All of the above~~.
    E. Some of the above

4. Nitrites are often added to meat products as preservatives. In a study of the effect of these chemicals on bacteria, the rate of uptake of a radio-labeled amino acid was measured for a number of cultures of bacteria, some growing in a medium to which nitrites had been added. Here are the summary statistics from this study.

Group	n	$\bar{x}$	s
Nitrite	30	7880	1115
Control	30	8112	1250

This is an example of :
    a. Matched pairs
    b. Two sample t test
    c. Two sample z test
    d. χ^2 goodness of fit
    e. none of these

Some problems from quiz 12:

$1-\alpha$ or $1-$pvalue

2) A one-sided significance test gives a P-value of .04. From this we can
a) Reject the null hypothesis with 95% confidence.
b) Reject the null hypothesis with 96% confidence. ← $1-.04=.96$
c) Say that the probability that the null hypothesis is false is .04.
d) Say that the probability that the null hypothesis is true is .04.

3&4) It is believed that the average amount of money spent per U.S. household per week on food is about \$98, with standard deviation \$10. A random sample of 100 households in a certain affluent community yields a mean weekly food budget of \$100. We want to test the hypothesis that the mean weekly food budget for all households in this community is higher than the national average. Are the results significant at the 5% level?

$z=\frac{100-98}{10/\sqrt{100}}=2$ pvalue: $p(z>2)=.02275<\alpha$

#3 asks for null and alternate hypothesis and #4 asks for reject or fail to reject

$H_0: \mu=98$ $H_a: \mu>98$ Rej RH_0

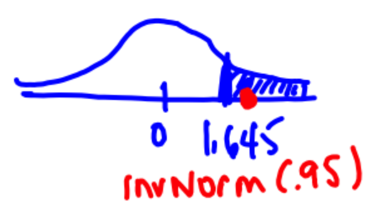
5&6) Based on information from a large insurance company, 67% of all damage liability claims are made by single people under the age of 25. A random sample of 53 claims showed that 42 were made by single people under the age of 25. Does this indicate that the insurance claims of single people under the age of 25 is higher than the national percent reported by the large insurance company?

#5 – State the null and alternate hypothesis.
#6 - Give the test statistic and your conclusion.

H_0 $p=.67$ $\hat{p}=\frac{42}{53}$ $n=53$

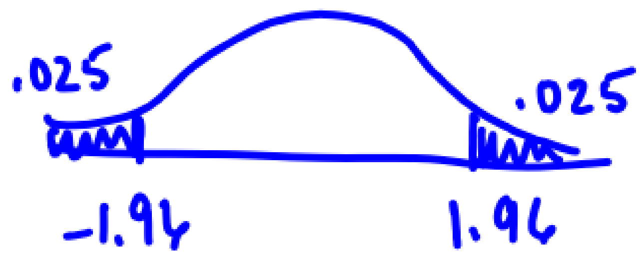
H_a $p>.67$

$$z=\frac{42/53-.67}{\sqrt{\frac{.67(.33)}{53}}}=\underline{1.896}$$

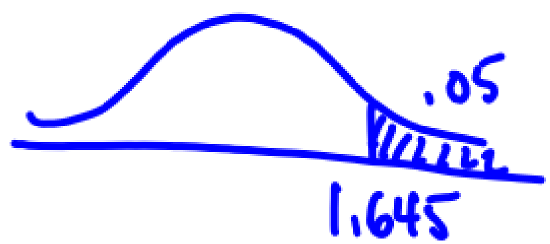


$\underline{RH_0}$

$H_a \neq \qquad \alpha = .05$



$H_a > \qquad \alpha = .05$



one sample t-test

7) Let x represent the hemoglobin count (HC) in grams per 100 milliliters of whole blood. The distribution for HC is approximately normal with $\mu = 14$ for healthy adult women. Suppose that a female patient has taken 12 laboratory blood samples in the last year. The HC data sent to her doctor is listed below. We would like to know if the data indicates this patient has significantly high HC compared to the population. $n=12$

(19, 14, 23, 20, 15, 19, 21, 16, 18, 18, 16, 21) → $\bar{x} = 18.333$, $s_x = 2.708$ (enter into L_1, stat - calc - one var stat)

#7 - State the null and alternate hypothesis. $H_0: \mu = 14$ $\quad H_a: \mu > 14$

#8 - Give the p-value and interpret the results.

$$t = \frac{18.333 - 14}{2.708/\sqrt{12}} = 5.543$$

p: tcdf(5.543, big, 11) (1 - pt(5.543, 11))

pvalue: 8.73×10^{-5} RH₀

9) An experimenter flips a coin 100 times and gets 44 heads. Test the claim that the coin is fair against the two-sided claim that it is not fair at the level $\alpha = .01$.

a) ~~H_o: $p = .5$, H_a: $p < .5$; $z = -1.20$; Reject H_o at the 1% significance level.~~

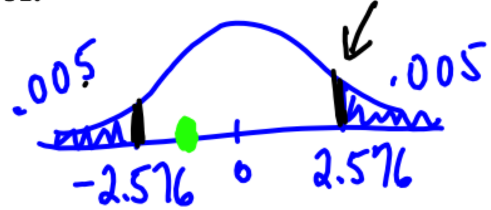
b) H_o: $p = .5$, H_a: $p \neq .5$; $z = -1.20$; Fail to reject H_o at the 1% significance level.

c) ~~H_o: $p = .5$, H_a: $p < .5$; $z = -1.21$; Fail to reject H_o at the 1% significance level.~~

d) ~~H_o: $p = .5$, H_a: $p \neq .5$; $z = -1.21$; Fail to reject H_o at the 1% significance level.~~

e) H_o: $p = .5$, H_a: $p \neq .5$; $z = -1.20$; Reject H_o at the 1% significance level.

invNorm (.995)



$\hat{p} = .44$ $\qquad p = .5$

$$z = \frac{.44 - .5}{\sqrt{\frac{.5(.5)}{100}}} = -1.2$$

$.44 - .5$

$-.06$

$\div \sqrt{(.5 * .5 \div 100)}$ >

pvalue: $2p(z < -1.2) = .23 > \alpha$

10) In a experiment on relaxation techniques, subject's brain signals were measured before and after the relaxation exercises with the following results:

Matched Pairs

Person	1	2	3	4	5
Before	31	38	66	51	32
After	26	35	57	51	26
bef - aft	5	3	9	0	6

> 0

$\bar{x} = 4.6$ $\quad$ $df = 4$

$s = 3.36$

Assuming the population is normally distributed, is there sufficient evidence to suggest that the relaxation exercise slowed the brain waves? (Use $\alpha=0.05$)

a) There is not enough information to make a conclusion.

b) Fail to reject the null hypothesis which states there is no change in brain waves.

c) Reject the null hypothesis which states there is no change in brain waves in favor of the alternate which states the brain waves slowed after relaxation.

$H_0: \mu_D = 0$ $\quad$ D = before - after

$H_a: \mu_D > 0$

$$t = \frac{4.6 - 0}{3.36/\sqrt{5}} = 3.06$$

pvalue: $p(t > 3.06) = .0188 < \alpha$

tcdf(3.06, 9999, 4) or 1 - pt(3.06, 4)

11 & 12) An auditor for a hardware store chain wished to compare the efficiency of two different auditing techniques. To do this he selected a sample of nine store accounts and applied auditing techniques A and B to each of the nine accounts selected. The number of errors found in each of techniques A and B is listed in the table below:

Matched pairs

$D = A - B$

$df = 8$

acct	Errors in A	Errors in B	D
1	45	31	14
2	48	37	11
3	46	39	11
4	48	37	11
	52	54	-2
	50	45	5
	49	49	0
	40	41	1
9	45	50	-5

$H_0: \mu_D = 0$

$H_a: \mu_D < 0$

$\bar{X}_D =$

$S =$

Does the data provide sufficient evidence to conclude that the number of errors in auditing technique A is fewer than the number of errors in auditing technique B at the 0.1 level of significance?

#11 - Select the [Alternative Hypothesis, Value of the Test Statistic]. (Hint: the samples are dependent)

#12 - Select the [Rejection Region, Decision of Reject (RH_0) or Failure to Reject (FRH_0)]. (Hint: the samples are dependent)

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5. Suppose a test of the hypotheses produces a P-value of 0.105. The correct action would be to
   a. Reject H_o
   b. Accept H_o
   c. Fail to reject H_o
   d. Accept H_a

R Studio: $H_a: \mu \neq \mu_o$

2(1 − pt(pos test stat, df)

2(pt(neg test stat, df))