

Math 2311

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Math 2311
Sections 3.1 – 3.2

Popper 03

Given the following sampling distribution:

X	0	1	2	3	4
P(X)	k	3k	2k	k	2k

1. What is the value of k ?

2. What is $P(X > 2)$?

Example: Suppose you are given the following distribution table:

X	1	2	3	4	5	6	7
P(X)	0.15	0.05	0.10	?	0.10	0.15	0.15

Find the following:

$P(X = 4)$

$P(X < 2)$

$P(2 < X \leq 5)$

$P(X > 3)$

What is the expected value?

The variance and standard deviation?

Rules for means and variances:

Suppose X is a random variable and we define W as a new random variable such that $W = aX + b$, where a and b are real numbers. We can find the mean and variance of W with the following formula:

$$E[W] = E[aX + b] = aE[X] + b$$

$$\sigma_W^2 = \text{Var}[W] = \text{Var}[aX + b] = a^2 \text{Var}[X]$$

Likewise, we have a formula for random variables that are combinations of two or more other independent random variables. Let X and Y be independent random variables,

$$E[X + Y] = E[X] + E[Y]$$

$$\sigma_{X+Y}^2 = \text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$$

and

$$E[X - Y] = E[X] - E[Y]$$

$$\sigma_{X-Y}^2 = \text{Var}[X - Y] = \text{Var}[X] + \text{Var}[Y]$$

#14 from text: Suppose you have a distribution, X , with mean = 22 and standard deviation = 3. Define a new random variable $Y = 3X + 1$.

a. Find the variance of X .

b. Find the mean of Y .

c. Find the variance of Y .

d. Find the standard deviation of Y .

From Quiz:

Suppose you want to play a carnival game that costs 8 dollars each time you play. If you win, you get \$100. The probability of winning is $3/100$. What is the expected value of the amount that you, the player, stand to gain?

A random sample of 2 measurements is taken from the following population of values: -2, -1, 1, 2, 5. What is the probability that the range of the sample is 6?

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Given the following sampling distribution:

X	-16	-13	-7	11	14
P(X)	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{7}{100}$	—

3. $P(X = 14) =$

4. What is the mean of this sampling distribution?

5. Suppose $Y = 3x + 2$. What is the mean of Y?

Section 3.2:

A **Bernoulli Trial** is a random experiment with the following features:

1. The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
2. The probability of success is p and probability of failure is $q = 1 - p$.

A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.

If we wish to keep track of the number of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**.

A **binomial experiment** occurs when the following conditions are met:

1. Each trial can result in one of only two mutually exclusive outcomes (success or failure).
2. There are a fixed number of trials.
3. Outcomes of different trials are independent.
4. The probability that a trial results in success is the same for all trials.

The random variable $X =$ number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials. The possible values of X are whole numbers that range from 0 to n . As an abbreviation, we say $X \sim B(n, p)$.

Binomial probabilities are calculated with the following formula:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = {}_n C_k p^k (1-p)^{n-k}$$

In R, $P(X = k) = \text{dbinom}(k, n, p)$.

With a TI-83/84 calculator, $P(X = k) = \text{binompdf}(n, p, k)$

Example: A fair coin is flipped 30 times.

What is the probability that the coin comes up heads exactly 12 times?

$$P(X \leq k) = \text{pbinom}(k, n, p)$$

$$P(X \leq k) = \text{binomcdf}(n, p, k)$$

What is the probability the coin comes up heads less than 12 times?

What is the probability the coin comes up heads more than 12 times?

The mean and variance of a binomial distribution are computed using the following formulas:

$$\mu = E[X] = np$$

$$\sigma^2 = np(1-p)$$

From text:

17. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

a. No one will contract the flu?

b. All will contract the flu?

c. Exactly two will get the flu?

d. At least two will get the flu?

e. Let X = number of family members contracting the flu. Create the probability distribution table of X .

f. Find the mean and variance of this distribution.

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6. In testing a new drug, researchers found that 6% of all patients using it will have a mild side effect. A random sample of 11 patients using the drug is selected. Find the probability that none will have this mild side effect.

7. Suppose you have a binomial distribution with $n = 20$ and $p = 0.4$. Find $P(8 \leq X \leq 12)$.