Math 2311

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4.3 - Standard Normal Calculations

Z-Score: $$z = \frac{x - \mu}{\sigma}$$

What does a z-score tell us?

What is the mean and standard deviation of the standard normal distribution?

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1. If a student's exam score corresponds to a negative z-score, then the student has a score that is less than the mean of the set of exam scores.

2. The z-score associated with a measurement x represents the number of _____ that x lies above the mean or below the mean.

Suppose family income in a particular suburb has been found to be approximately normal with a mean of $52,137 and a standard deviation of $19,452. What percentage has an income in the range of $50,000 to $80,000?

Now, let's suppose we know the percentile rank or the probability and want to find the corresponding z-score.

We can use Table A and look up the percentile (remember, it shows the area to the left) or we can use the command invNorm(percent) on the TI or qnorm(percent) in R.

Example: Find the value of c so that

A. $P(Z < c) = 0.8810$

B. $P(Z > c) = 0.1469$

C. $P(-c < Z < c) = 0.7699$

Another example: Suppose you rank in the top 10% of your class. If the mean gpa is 2.8 and the standard deviation is 0.43, what is your gpa? (Assume a normal distribution)

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3. If a sample has a mean of 48 and a standard deviation of 3.2, what is the value in the set that corresponds to a z-score of -1.2?

4. Find $P(-1.9 < Z < 1.2)$

5. Find c such that $P(Z > c) = 0.8790$

Section 4.4 – The distributions of $\overline{x}$ and $\hat{p}$

We will start this section by looking at the "sampling distribution of the means" for a set of data – this is the same as the "distribution of the sample means".

Consider the population consisting of the values 3,5,9,11 and 14

μ = __________ σ = ____________

Let's take samples of size 2 from this population.

The set above is the **sampling distribution of size 2** for this population.

Let's take samples of size 3 from this population.

The set above is the **sampling distribution of size 3** for this population.

List all the possible pairs/triples from 3,5,9,11 and 14 and find their means.

pairs	$\overline{x}$

$\mu_{\overline{x}} =$ _____, $\sigma_{\overline{x}} =$ ______

triples	$\overline{x}$

$\mu_{\overline{x}} =$ _____, $\sigma_{\overline{x}} =$ _____

Compare $\mu_{\overline{x}}$ (the mean of the sample means) to μ.

What do you notice about $\sigma_{\overline{x}}$?

Suppose that $\bar{x}$ is the mean of a simple random sample of size *n* drawn from a large population. If the population mean is μ and the population standard deviation is σ, then the mean of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu$ and the standard deviation of the sampling distribution is $\sigma_{\bar{x}} = \sigma / \sqrt{n}$. If our original population has a normal distribution, the sample mean's distribution is $N(\mu, \sigma / \sqrt{n})$.

An unbiased statistic is a statistic used to estimate a parameter in such a way that mean of its sampling distribution is equal to the true value of the parameter being estimated. We consider the above values to be unbiased estimates of our distribution.

The Central Limit Theorem states that if we draw a simple random sample of size *n* from any population with mean μ and standard deviation σ, when *n* is large the sampling distribution of the sample mean $\bar{x}$ is close to the normal distribution $N(\mu, \sigma / \sqrt{n})$.

Determining whether *n* is large enough for the central limit theorem to apply depends on the original population distribution. The more the population distribution's shape is from being normal, the larger the needed sample size will be. A rule of thumb is that $n > 30$ will be large enough.

Examples:

Suppose a random sample of 90 measurements is selected from a population with a mean of 35 and a variance of 300. Find the mean and standard deviation of this sampling distribution of $\overline{x}$.

A random sample of 400 32-ounce cans of fruit nectar is drawn from among all cans produced in a run. Prior experience has shown that the distribution of the contents has a mean of 32 ounces and a standard deviation of 0.32 ounce. What is the probability that the mean contents of the 400 sample cans is less than 31.984 ounces?

Current research indicates that the distribution of the life expectancies of a certain protozoan is normal with a mean of 48 days and a standard deviation of 10.2 days. Find the probability that a simple random sample of 49 protozoa will have a mean life expectancy of 49 or more days.

Suppose that a random sample of size 64 is to be selected from a population with mean 42 and standard deviation 9.

What is the approximate probability that $\overline{x}$ will be within 0.5 of the population mean?

What is the approximate probability that $\overline{x}$ will be more than 0.5 away from the population mean?

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A waiter estimates that his average tip per table is $20 with a standard deviation of $4. If we take samples of 9 tables at a time, calculate the following probabilities when the tip per table is normally distributed.

6. What is the probability that his average tip is more than $19?

7. What is the probability that his average tip is between $19 and $22?

Sampling proportions:

When X is a binomial random variable (with parameters n and p) the statistic $\hat{p}$, the sample proportion, is equal to $\frac{X}{n}$. The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$. If our population size is at least 10 times the sample size, the standard deviation of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$. We can use the normal approximation for the sampling distribution of $\hat{p}$ when $np \geq 10$ and $n(1-p) \geq 10$.

Examples:

In a large population, 72% of the households have cable tv. A simple random sample of 100 households is to be contacted and the sample proportion computed.

What is the mean and standard deviation of the sampling distribution of the sample proportions?

What is the probability that the sampling distribution of sample porportions is less than 73%?

A large high school has approximately 1,200 seniors. The administration of the school claims that 82% of its graduates are accepted into colleges. If a simple random sample of 100 seniors is taken, what is the mean and the standard deviation of the sampling distribution?

What is the probability that at most 64 of them will be accepted into college?

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In a large population, 86% of the households own computers. A simple random sample of 100 households is to be contacted and the sample proportion computed.

8. What is the mean and standard deviation of the sampling distribution of the sample proportions?

9. What is the probability $\hat{p}$ will be between 82% and 90%?