## Math 2311

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Let's continue our look at Bivariate Data with this example:
Suppose we want to know if there is an association between the number of spaces a property is from GO and the cost of the property in a monopoly game. The data is below:

| Property | Spaces from GO | Cost |
| :--- | :--- | :--- |
| Mediterranean Avenue | 1 | 60 |
| Baltic Avenue | 3 | 60 |
| Reading Railroad | 5 | 200 |
| Oriental Avenue | 6 | 100 |
| Vermont Avenue | 8 | 100 |
| Connecticut Avenue | 9 | 120 |
| St. Charles Place | 11 | 140 |
| Electric Company | 12 | 150 |
| States Avenue | 13 | 140 |
| Virginia Avenue | 14 | 160 |
| Penn Railroad | 15 | 200 |
| St. James Place | 16 | 180 |
| Tennessee Avenue | 18 | 180 |
| New York Avenue | 19 | 200 |
| Kentucky Avenue | 21 | 220 |
| Indiana Avenue | 23 | 220 |
| Illinois Avenue | 24 | 240 |
| B \& O Railroad | 25 | 200 |
| Atlantic Avenue | 26 | 260 |



| Ventnor Avenue | 27 | 260 |
| :--- | :--- | :--- |
| Water Works | 28 | 150 |
| Marvin Gardens | 29 | 280 |
| Pacific Avenue | 31 | 300 |
| North Carolina Avenue | 32 | 300 |
| Pennsylvania Avenue | 34 | 320 |
| Short Line Railroad | 35 | 200 |
| Park Place | 37 | 350 |
| Boardwalk | 39 | 400 |

Which variable is explanatory and which is response?
Enter your data and create a scatterplot.


What can be said of the scatterplot?

The correlation coefficient measures the strength and direction of the linear relationship between two quantitative variables. The formula to find $r$ is:
$r=\frac{1}{n-1} \sum\left(\frac{x_{i}-\bar{x}}{s_{x}}\right)\left(\frac{y_{i}-\bar{y}}{s_{y}}\right)$
The point $(\bar{x}, \bar{y})$ is:
The values of $s_{x}$ and $s_{y}$ are the individual standard deviations of $x$ and $y$ respectively. $n$ represents the number of data pieces.

## Facts about Correlation:

1. Positive $r$ indicates positive association and negative $r$ indicates negative association between variables.
2. $r$ is always between -1 and 1 .
3. The closer $|r|$ is to 1 , the stronger the association. A weak association will have an $r$ value close to 0 .
4. Correlation is strongly influenced by outliers.

Using the Monopoly data, we can find the correlation coefficient with R using the command

```
cor(spaces, cost).
>cor(spaces,cost)
[1] 0.8779736
```

With the TI-83/84 calculator, we need to take a few more steps:

1. We must make sure the diagnostics is turned on:

Do this by clicking 2ND - CATALOG and scroll down to Diagnostics
2. Choose STAT - CALC then $4: \operatorname{LinReg}(a x+b)$
3. Make sure your Xlist is L1 and Ylist is L2 and select Calculate:


What does this value reveal about our data from the Monopoly example?

## Popper 10

1. If your computed correlation coefficient $\mathrm{r}=+1.2$, then you have better than a perfect positive correlation.
2. You should expect that there is a positive correlation between the age of your computer and its resale value.

A regression line is a line that describes the relationship between the explanatory variable $x$ and the response variable $y$. Regression lines can be used to predict a value for $y$ given a value of $x$.

The least squares regression line (or LSRL) is a mathematical model used to represent data that has a linear relationship. We want a regression line that makes the vertical distances of the points in a scatter plot from the line as small as possible.


The least squares regression line formula is $\hat{y}=a+b x$
The slope, $b$ is calculated using $b=r\left(\frac{s_{y}}{s_{x}}\right)$ and the $y$-intercept is $a=\bar{y}-b \bar{x}$.

To calculate the values of $a$ and b for the regression line with R-Studio, we use the command $>\operatorname{lm}(\mathrm{y} \sim \mathrm{x})$

For the Monopoly problem from section 5.1, we have:
$>$ regline $=\operatorname{lm}($ cost $\sim$ spaces $)$
$>$ regline
Call:
$\operatorname{lm}$ (formula $=$ cost $\sim$ spaces)
Coefficients:
(Intercept) spaces
$67.283 \quad 6.784$
So, the regression line is $y=6.794 x+67.283$.

Note that I assigned a name to the 1 m command, this is not required unless you wish to use it again. We will use it again to plot the regression line on top of the scatterplot. The command is abline. $>$ abline(regline)

Now we can see how well the model fits the graph.


With the TI-83/84 we will follow some of the steps from section 5.2 with one difference. When we choose STAT - CALC.

If you are using a TI-84 Plus C, you will enter Y1 where it says Stor RegEQ on the LinReg screen

With the other TI-83/84 version, we will choose $\operatorname{LinReg}(\mathrm{ax}+\mathrm{b}) \mathrm{L} 1, \mathrm{~L} 2, \mathrm{Y} 1$
You select Y1 from VARS - Y-VARS
Now go to graph and graph the function. You may need to choose ZoomStat again.


The LSRL can be used to predict values of $y$ given values of $x$.
Let's use our model to predict the cost of a property 50 spaces from GO.

We need to be careful when predicting. When we are estimating $y$ based on values of $x$ that are much larger or much smaller than the rest of the data, this is called extrapolation.

Notice that the formula for slope is $b=r\left(\frac{s_{y}}{s_{x}}\right)$, this means that a change in one standard deviatior in $x$ corresponds to a change of $r$ standard deviations in $y$. This means that on average, for each unit increase in $x$, then is an increase (or decrease if slope is negative) of $|b|$ units in $y$.

Interpret the meaning of the slope for the Monopoly example.

The square of the correlation $(r), r^{2}$ is called the coefficient of determination. It is the fraction of the variation in the values of $y$ that is explained by the regression line and the explanatory variable.

When asked to interpret $r^{2}$ we say, "approximately $r^{2} * 100 \%$ of the variation in $y$ is explained by the LSRL of $y$ on $x$."

Facts about the coefficient of determination:

1. The coefficient of determination is obtained by squaring the value of the correlation coefficient.
2. The symbol used is $r^{2}$
3. Note that $0 \leq r^{2} \leq 1$
4. $r^{2}$ values close to 1 would imply that the model is explaining most of the variation in the dependent variable and may be a very useful model.
5. $r^{2}$ values close to 0 would imply that the model is explaining little of the variation in the dependent variable and may not be a useful model.

Interpret $r^{2}$ for the Monopoly problem.

Any questions on homework or quizzes??

## Popper 10

3. A least-squares regression line was fitted to the weights (in pounds) versus age (in months) of a group of many young children. The equation of the line is $\hat{y}=16.6+0.65 t$. Predict the weight of the child at 20 months.
4. A wildlife biologist is interested in the relationship between the number of chirps per minute for crickets $(y)$ and temperature. Based on the collected data, the least-squares regression line is $\hat{y}=10.53+3.41 x$, where x is the number of degrees Fahrenheit by which the temperature exceeds $50^{\circ}$. Which of the following best describes the meaning of the slope of the least-squares regression line?

Match the following scatter plots with the appropriate correlations from the list:


