## Math 2311

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## Popper 20

1. The $p$-value of a test of significance is the probability that:
2. If we want to claim that a population parameter is different from a specified value, this situation can be considered as a one-tailed test.

## 8.2 - Inference for a Population Proportion

For these inferences, $p_{0}$ represents the given population proportion.
$H_{0}: p=p_{0}$
$H_{a}: p \neq p_{0}$ or $p<p_{0}$ or $p>p_{0}$
Conditions:

1. The sample must be an SRS from the population of interest.
2. The population must be at least 10 times the size of the sample.
3. The number of successes and the number of failures must each be at least 10 (both $h \hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$ ).

Recall, the statistic used for proportions is

$$
\hat{p}=\frac{\# \text { of successes }}{\# \text { of observations }}
$$

For tests involving proportions that meet the above conditions, we will use the $z$ - test statistic
where

$$
z=\frac{\hat{p}-p_{o}}{\sqrt{\frac{p_{o}\left(1-p_{o}\right)}{n}}}
$$

## Examples:

3. A new shampoo is being test-marketed. A large number of 16 -ounce bottles were mailed out at random to potential customers in the hope that the customers will return an enclosed questionnaire. Out of the 1,000 returned questionnaires, 575 indicated that they like the shampoo and will consider buying it when it becomes available on the market. Perform a hypothesis test to determine if the proportion of potential customers is more than $50 \%$.
4. Mars Inc. claims that they produce M\&Ms with the following distributions:

| Brown | $30 \%$ | Red | $20 \%$ | Yellow | $20 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Orange | $10 \%$ | Green | $10 \%$ | Blue | $10 \%$ |

A bag of M\&Ms was randomly selected from the grocery store shelf, and the color counts were:

| Brown | 14 | Red | 14 | Yellow | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Orange | 7 | Green | 6 | Blue | 10 |

Conduct an appropriate test of the manufacturer's claim for the proportion of brown M\&Ms.

Decision errors can occur when choosing to reject or failing to reject the null hypothesis. There are two types of decision errors; Type I and Type II.
A Type I error occurs when you reject the null hypothesis when in fact it is true. This error would correspond to taking action when you would have been better off not doing so. A Type II error happens when the null hypothesis is accepted (fail to reject) when in fact the alternative hypothesis is true. The Type II error would correspond to taking no action when you would have been better off taking action. The power of a test against an alternative hypothesis is the probability that a fixed level, $\alpha$, significance test will reject the null hypothesis when that particular alternative is true. The following table can help to identify the types of errors.

|  | $H_{0}$ is true | $H_{\mathrm{a}}$ is true |
| :--- | :--- | :--- |
| Reject $H_{0}$ | Type I error | Decision is correct |
| Fail to reject $H_{0}$ | Decision is correct | Type II error |

## Example:

You are considering whether or not to play the lottery with your favorite numbers. What situations denote a type I and II errors?

|  | $H_{0}$ is true (Won't win) | $H_{\mathrm{a}}$ is true (Would win) |
| :--- | :--- | :--- |
| Reject $H_{0}$ (buy a ticket) |  |  |
| Fail to reject $H_{0}$ (don't buy) |  |  |
|  |  |  |

## Example:

A manufacturer of circuits had observed that, on average, $p=0.03$ of its circuits failed. One of the engineers suggests changes in the design to try to improve this percentage.

It is decided that $n=100$ circuits would be made using her method. The company will adopt her method if only zero or one of the circuits failed.

Identify the null and alternative hypotheses at play here. What are the possible Type I and Type II errors the company might make with this strategy?

What is the probability that the company will make a Type I error?

If the actual value for the new method is $p=0.01$, what is the probability that the company will make a Type II error?

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Suppose that the state said that $65 \%$ of all state residents are in favor of a lottery. An SRS of 100 people showed that 52 of those 100 surveyed are in favor of the lottery. Test the claim that the percent in favor of the lottery is less than $60 \%$.
3. The null and alternate hypothesis are:
4. What is the test statistic?
5. What is the p -value?

