(ehw 4)

A distribution of grades in an introductory statistics class (where A = 4, B = 3, etc) is:

X	0	1	2	3	4
P(X)	.10	.25	.30	??	.10

.25 =

.1 + .25 + .3 + x + .1 = 1

.75 + x = 1

x = .25

5. Find the lowest grade X_0 such that $P(X \geq X_0) < 0.5$

   a. 4
   b. 3
   c. 2
   d. 1
   e. none of these

$P(X \geq 4) = .10 < .5$

$P(X \geq 3) = .35 < .5$

$P(X=3) + P(X=4)$

$P(X \geq 2) = .65$ not $< .5$

$P(X=2) + P(X=3) + P(X=4)$

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Binomial $n = 100$ $p = .8$

A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

11. What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief?
    a. 0.0993
    b. 0.8663
    c. 0.5398
    d. 0.9102
    e. None of these

$P(75 \le X \le 90)$

subtract

0 1 2 ..... 73 74 | 75 76 .... 89 90 | 91 ... 100

$P(X \le a) = \text{binomcdf}(n, p, a)$ or $\text{pbinom}(a, n, p)$

$P(X \le 90) = \text{pbinom}(90, 100, .8)$

subtract

$P(X \le 74) = \text{pbinom}(74, 100, .8)$

when we subtract, leaves us with 75 ... 90

$\star\ P(a \le X \le b) = P(X \le b) - P(X \le a-1)\ \star$

A quarter back completes 60% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass.

Geom. $p = .6$

15. What is the probability that he attempts 4 or fewer passes before he completes one?
    a. 0.0256
    b. 0.9898
    c. 0.9744
    d. 0.0102
    e. none of these

Completes on 5th, 4th, 3rd, ...

The probability that the first success occurs on the n^{th} trial is

$$P(X = n) = (1-p)^{n-1} p$$

where p is the probability of success.

geometcdf(.6,5) = P(X ≤ 5)

pgeom(4,.6) = P(X ≤ 5)

= 1 − (.4)^5

$P(X = 5) = (.4)^4 (.6)$

Binomial vs. Geometric

Prob. of success
same for each trial (p) ← never changes

indep. trials

Know n (total trials)

dont know n

The probability that it takes *more* than n trials to see the first success is

$$P(X > n) = (1-p)^n$$

Complement of above would be it takes ≤ more than 5

$P(X \leq a)$ = geometcdf (p, a)
pgeom (a-1, p)

$P(X = a)$ = geometpdf (p, a)
dgeom (a-1, p)

} Geometric

$P(X \leq a)$ = binomcdf (n, p, a)
pbinom (a, n, p)

$P(X = a)$ = binompdf (n, p, a)
dbinom (a, n, p)

} Binomial

3.1

8. A box contains four slips of paper marked 1, 2, 3, and 4. Two slips are selected without replacement. Make a Probability Distribution for X, if X = the sum of the two numbers.

3 4 5

(1,2) (1,3) (1,4) ~~(3,1)~~

(2,3) (2,4) (3,4)

5 6 7

	1	2	3	4
1	X	3	4	5
2	3	X	5	6
3	4	5	X	7
4	5	6	7	X

12

$P(\text{sum} = 3) = 1/6 = 2/12$

sum	3	4	5	6	7
P(X)	1/6	1/6			

12. In a litter of seven kittens, three are female. You pick two kittens at random.

a. Create a probability model for the number of **male** kittens you get.

b. Find the expected number of **males**.

c. Find the standard deviation for your distribution.

3 F 4 m 7 total

mm MF FF

2 1 0 males

$\sqrt{Var}$ $Var = E[X^2] - (E[X])^2$

a)

X	0	1	2
P(X)	$\frac{{}_4C_0 \cdot {}_3C_2}{{}_7C_2}$		

X^2 0 1 4

same prob.

$E[X^2] = 0 \cdot (\ ) + 1(\ ) + 4(\ )$

sample space $= {}_7C_2$

0 males from 4 and 2 females from 3

Prob of 1 male $= \frac{{}_4C_1 \cdot {}_3C_1}{{}_7C_2}$

3.3

12. Of the automobiles produced at a particular plant, 40% had a certain defect.

a. What is the probability that more than 50 cars will need to be inspected before one with the defect is found?

b. What is the probability that the twentieth car inspected will have a defect?

c. Suppose a company purchases five of these cars. What is the probability that exactly one of the five cars has a defect?

$p = .4$

a) $P(X > 50) = (1 - .4)^{50}$

b) $P(X = 20)$

c) now its binomial $n = 5$

$P(X = 1) = \text{binompdf}(5, .4, 1)$

Q3

Question 9

In testing a certain kind of missile, target accuracy is measured by the average distance X (from the target) at which the missile explodes. The distance X is measured in miles and the sampling distribution of X is given by:

X	0	10	50	100
P(X)	$1/34$	$1/17$	$2/17$	$27/34$

Calculate the variance of this sampling distribution.

a) 29.4

b) 288.5

c) 4873.6

d) 85.9

e) 865.4

f) None of the above

$$E[X] = 0(1/34) + 10(1/17) + 50(2/17) + 100(27/34)$$

$$E[X^2] = 0(1/34) + 10^2(1/17) + 50^2(2/17) + 100^2(27/34)$$

$$Var = E[X^2] - (E[X])^2$$

$$E[X] = 85.882352\,94$$

$$E[X^2] = 8241.176471$$

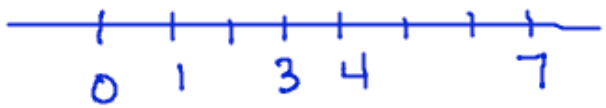
$$Var[X] = 8241.176 - (85.88)^2$$

$$= 865.8$$

Question 12

A random sample of 2 measurements is taken from the following population of values: 0, 1, 3, 4, 7. What is the probability that the range of the sample is 6?

range = max - min



a) 0.5

b) 0.2

c) 0.4

d) 0.1

e) 0.3

f) None of the above

(0, 1) = 1

(0, 3) = 3

(0, 4) = 4

(0, 7) = 7

(1, 3) = 2

(1, 4) = 3

(1, 7) = 6

(3, 4) = 1

(3, 7) = 4

(4, 7) = 3

$\frac{1}{10}$

Question 13

A furniture store is having a sale on sofas and you're going to buy one. The advertisers know that buyers get to the store and that 1 out of 4 buyers change to a more expensive sofa than the one in the sale advertisement. Let X be the cost of the sofa. What is the average cost of a sofa if the advertised sofa is $300 and the more expensive sofa is $450?

a) 330.00

b) 337.50

X	300	450
P(X)	3/4	1/4

average $\rightarrow E[X] = 300(3/4) + 450(1/4)$

10)

X	+$5	−$95
P(X)	97/100	3/100

← carnival gains

$E[X] = 5(97/100) - 95(3/100)$

Question 10

Suppose you want to play a carnival game that costs 5 dollars each time you play. If you win, you get $100. The probability of winning is $3/100$. What is the expected value of the amount the carnival stands to gain?

Question 8

In testing a certain kind of missile, target accuracy is measured by the average distance X (from the target) at which the missile explodes. The distance X is measured in miles and the sampling distribution of X is given by:

X	0	10	50	100
P(X)	1/40	1/20	1/10	33/40

Calculate the mean of this sampling distribution.

$$E[x] = 0\left(\tfrac{1}{40}\right) + 10\left(\tfrac{1}{20}\right) + 50\left(\tfrac{1}{10}\right) + 100\left(\tfrac{33}{40}\right)$$

You did not answer the question.

Among 6 electrical components exactly one is known not to function properly. If 4 components are selected randomly, find the prob that exactly one does not function properly.

1 doesnt function properly
5 function properly

$$n(S) = {}_6C_4$$

$$P(1 \text{ doesnt function}) = \frac{{}_1C_1 \cdot {}_5C_3}{{}_6C_4}$$

$$P(\text{all function}) = \frac{{}_5C_4}{{}_6C_4}$$

Q2 #15

The probability that a randomly selected person has high blood pressure (the event H) is $P(H) = 0.4$ and the probability that a randomly selected person is a runner (the event R) is $P(R) = 0.5$. The probability that a randomly selected person has high blood pressure and is a runner is 0.1. Select the false statement.

a) $P(R^c \cup H^c) = 0.9$

b) $P(H \cap R^c) = 0.3$

c) $P(R \cup H) = 0.8$ T

d) H and R are independent events. F

e) H and R are not mutually exclusive. T

$P(H \cup R) = .8$

$P(H)P(R) = P(H \cap R)$

$(.4)(.5) \neq .1$

If mutually exclusive $\Rightarrow P(H \cap R) = 0$

Suppose a card is drawn from a deck of 52 playing cards. What is the probability of drawing a 5 or a king?

a) $\frac{1}{13}$

$$P(K) = \frac{4}{52} = \frac{1}{13}$$

$$+$$

$$P(5) = \frac{4}{52} = \frac{1}{13}$$

b) $\frac{1}{156}$

c) $\frac{1}{4}$

Q3 #7

Question 7

Suppose you have a distribution, X, with mean = 29 and standard deviation = 6. Define a new random variable Y = 4X - 5. Find the mean and standard deviation of Y.

a) E[Y] = 111; σY = 19

b) E[Y] = 116; σY = 96

c) E[Y] = 111; σY = 96

d) E[Y] = 116; σY = 19

e) E[Y] = 111; σY = 24

$E[X] = 29 \quad \sigma_X = 6 \quad VAR[X] = 36$

$Y = 4X - 5$

$E[Y] = E[4X-5] = 4E[X] - 5$

$VAR[Y] = 4^2 \, VAR[X]$

$\sigma_y = 4 \cdot \sigma_x = 4(6)$

A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

6. Define the random variable being measured
   a. X= the number of people who have headaches cured
   b. X= the number of people who take the remedy
   c. X= the number of selected patients
   d. None of these

X

P(X)

c. P(select 5 patients) ??

b. P(5 people get the remedy)??

a. P(5 people have headache cured)??

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

13. $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\quad}{.37}$