Among 16 electrical components exactly 3 are known not to function properly. If 7 components are randomly selected, find the following probabilities:
(i) The probability that all selected components function properly.
(ii) The probability that exactly 2 are defective.
(iii) The probability that at least 1 component is defective.

0 1 2 3
defectives

16 - total
choosing 7 ⇒ sample space = $\boxed{{}_{16}C_7}$

→ 3 do not function
13 do function

i) $P(\text{all 7 function}) = \dfrac{{}_{13}C_7}{{}_{16}C_7}$ ⇐ 0 defective

ii) $P(\text{2 defective}) = \dfrac{{}_3C_2 \cdot {}_{13}C_5}{{}_{16}C_7}$

iii) P(at least one defective) = 1 − P(0 defect)
$\boxed{1, 2, 3}$
1 − answer to part i

complement of 0 defectives

Public Page

3 T/F → definitions, concepts
4 M/C → like PT
} 8pts

3 F/R → like PT or Review Sheet → 14 or 16

50 minutes

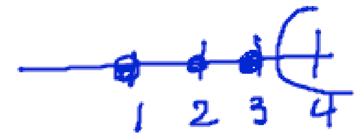
for free response round at least to hundredths place ehw4

13. How many passes can the quarterback $\boxed{\text{expect}}$ to throw before he completes a pass? (Round to nearest whole number)
   a. 2
   b. 10
   c. 3
   d. 6
   e. none of these

$E[X]$ or mean

$\frac{1}{p} = \frac{1}{.6} = \frac{10}{6} = 1\frac{2}{3} \approx 2$

14. Determine the probability that it takes more than 3 attempts before he completes a pass.
   a. 0.0384
   b. 0.096
   c. 0.064
   d. 0.0256
   e. none of these



$P(X > 3) = 1 - P(X \le 3)$

$1 - \text{geometcdf}(.6, 3)$

$1 - \text{pgeom}(2, .6)$

$3-1$

15. What is the probability that he attempts 4 or fewer passes before he completes one?
   a. 0.0256
   b. 0.9898
   c. 0.9744
   d. 0.0102
   e. none of these

$P(X \le 4) = \text{geometcdf}(.6, 4)$

$\text{pgeom}(3, .6)$

A quarter back completes 60% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass.

Geometric (no n)

$p = .6$

Geometric $P(X > n) = (1-p)^n$

31. A psychologist interested in right-handedness versus left-handedness and in IQ scores collected the following data from a random sample of 2000 high school students.

	Right-handed	Left-handed	Total
High IQ	190	10	200
Normal IQ	1710	90	1800
Total	1900	100	2000

$P(L) = \frac{100}{2000} = .05$

a. What is the probability that a student from this group has a high IQ? $200/2000 = .1$

b. What is the probability that a student has a high IQ given that she is left-handed?

c. Are high IQ and left-handed independent? Why or why not?

b) $P(H|L) = \frac{10}{100} = .1$

$P(H) \cdot P(L) = P(H \cap L)$

$(.1)(.05) = 10/2000$

$.005 = .005$ ✓

Be able to do poppers like $P(E) = \_\_$ $P(F) = \_\_$ $P(E \cup F) = \_\_$

Find $P(E \cap F)$, $P(E|F)$, ...

Question 4

The probability that a randomly selected person has high blood pressure (the event H) is $\boxed{P(H) = 0.2}$ and the probability that a randomly selected person is a runner (the event R) is $\boxed{P(R) = 0.4.}$ The probability that a randomly selected person has high blood pressure and is a runner is 0.1. Find the probability that a randomly selected person either has high blood pressure or is a runner or both.

$P(H \cap R) = .1$

a) 0.6

b) 0.9

$P(H \cup R) = P(H) + P(R) - P(H \cap R)$

c) 0.4

$= .2 + .4 - .1$

d) 0.5

$= .5$

e) 0.8

f) None of the above.

Question 9

Given the following sampling distribution:

X^2: 0, 16, 36, 121, 256

X	0	4	6	11	16
P(X)	3/100	1/20	7/100	1/20	.8

(i) What is the mean of this sampling distribution?
(ii) What is the standard deviation of this sampling distribution?

$$\frac{3}{100} + \frac{5}{100} + \frac{7}{100} + \frac{5}{100} + x = 1$$

$$\frac{20}{100} + x = 1$$

$$x = \frac{80}{100} = .8$$

a) (i) 7.40 (ii) 4.36

b) (i) 0.42 (ii) 4.36

c) (i) 0.42 (ii) 19.01

d) (i) 13.97 (ii) 4.36

e) (i) 13.97 (ii) 19.01

f) None of the above

$$E[x] = \mu = 0\left(\frac{3}{100}\right) + 4\left(\frac{1}{20}\right) + 6\left(\frac{7}{100}\right) + 11\left(\frac{1}{20}\right) + 16(.8)$$

$$= 13.97$$

$$Var[x] = E[x^2] - (E[x])^2$$

$$E[x^2] = 0(3/100) + 16(1/20) + 36(7/100) + 121(1/20) + 256(.8)$$

$$Var = 214.17 - (13.97)^2 = 19.0091$$

$$\sigma_x = \sqrt{19.0091} = 4.36$$

Public Page 5

Question 10

Suppose you have a distribution, X, with mean = 10 and standard deviation = 3. Define a new random variable Y = 5X - 5. Find the mean and standard deviation of Y.

a) E[Y] = 50; σY = 10

b) E[Y] = 50; σY = 75

c) E[Y] = 45; σY = 75

d) E[Y] = 45; σY = 15

e) E[Y] = 45; σY = 10

f) None of the above

$$E[5X-5] = 5E[X]-5 = 5(10)-5 = 45$$

$$\sigma[5X-5] = 5\sigma_X = 5(3) = 15$$

$$E[aX+b] = aE[X]+b$$

$$Var[aX+b] = a^2 Var[X]$$

$$\sigma[aX+b] = a\sigma_X$$

22. Twelve people were asked how many movies they saw last month. The results were:

2 6 1 3 4 2 1 5 3 6 4 5

a. Find the mean and median.
b. Find the variance and the standard deviation.
c. Based on the values you get, what can you say about the shape of the distribution of the data set? Explain briefly.
d. Find the five-number summary.
e. Determine the interval for outliers.

min = 1 max = 6

Q1 = 2 Q2 = 3.5 Q3 = 5

IQR = 5 - 2 = 3

1.5 IQR = 1.5(3) = 4.5

Q1 - 4.5 = -2.5

Q3 + 4.5 = 9.5

[-2.5, 9.5]

no outliers

27. Suppose $P(A)=0.72$, $P(B)=0.46$ and $P(A \cup B)=0.86$

a. Find $P(A \cap B)$
b. Find $P(A \mid B)$
c. Find $P(B \mid A)$
d. Area A and B independent?

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.86 = .72 + .46 - x$

$.86 = 1.18 - x$

$x = 1.18 - .86 = .32$

$P(A \cap B) = .32$

b. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.32}{.46} = .6957$

c. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.32}{.72} = .4444$

d. $P(A)P(B) \stackrel{?}{=} P(A \cap B)$ <u>NO</u>

$(.72)(.46) \neq .32$

24. STATISTICS

S - 3

T - 3

A - 1

I - 2

C - 1

$$\frac{10!}{3!\,3!\,1!\,2!\,1!} = \frac{10!}{3!\,3!\,2!} = 50,400$$

$$P = \underbrace{\frac{n!}{r!\,s!\,t!}}_{\text{repeats}}$$

Permutation - arrangement
order matters

Combination - order doesnt matter
choosing 7 lightbulbs from 13

Question 8

Given the following sampling distribution:

X	-20	-13	-3	4	20
P(X)	$^3/_{100}$	$^1/_{50}$	$^9/_{100}$	$^1/_{100}$	___

$\underbrace{\qquad}_{15/100}$ $85/100 = .85$

$P(X=20)$

(i) Find P(X = 20)
(ii) Find P(X > -13) $= \frac{9}{100} + \frac{1}{100} + \frac{85}{100} = \frac{95}{100}$

$= .95$

a) (i) 0.85 (ii) 0.87

b) (i) 0.95 (ii) 0.84

c) (i) 0.20 (ii) 0.20

d) (i) 0.85 (ii) 0.95

e) (i) 0.84 (ii) 0.86

f) None of the above

A

26. Suppose that 58% of all customers of a large insurance agency have automobile policies with the agency, 42% have homeowner's policies, and 23% have both. What is the probability that the customer has at least one of the policies?

$P(A) = .58 \quad P(H) = .42 \quad P(A \cap H) = .23$

one or the other or both

$P(A \cup H) = P(A) + P(H) - P(A \cap H)$

$= .58 + .42 - .23$

33. A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

a. What kind of distribution does X have? (Binomial or Geometric?)
b. Calculate the mean and standard deviation of X.
c. Determine the probability that exactly 80 subjects experience headache relief with this remedy.
d. What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief?

a. $\begin{cases} n = 100 \\ p = .8 \\ \text{independ. trials} \end{cases}$

mean $= np = 100(.8) = 80$

b. $\sigma = \sqrt{np(1-p)} = \sqrt{100(.8)(.2)} = 4$

c. $P(X = 80) = \left.\begin{array}{l} \text{binompdf}(100, .8, 80) \\ \text{dbinom}(80, 100, .8) \end{array}\right\} = .099$

d. $P(75 \le X \le 90) = P(X \le 90) - P(X \le 74)$

binomcdf(100, .8, 90) - binomcdf(100, .8, 74)

pbinom(90, 100, .8) - pbinom(74, 100, .8)

.9102

23. Suppose that from a group of 9 men and 8 women, a committee of 5 people is to be chosen.

a. In how many ways can the committee be chosen?
b. In how many ways can the committee be chosen so that there are exactly 3 men and 2 women?
c. What is the probability that the committee has exactly 3 men and 2 women?

9 men 8 women $\Rightarrow$ 17 total

Choosing 5

a. ${}_{17}C_5 = 6188$

b. ${}_{9}C_3 \cdot {}_{8}C_2 = 84 \cdot 28 = 2352$

c. $P(3\text{ men } \& \text{ } 2\text{ women}) = \dfrac{2352}{6188}$

34. A basketball player completes 64% of her free-throws. We want to observe this player during one game to see how many free-throw attempts she makes before completing one.

a. What type of distribution is this?
b. What is the probability that the player misses 3 free-throws before she has makes one? $\rightarrow (.36)(.36)(.36)(.64)$
c. How many free-throw attempts can the player expect to throw before she gets a basket?
d. Determine the probability that it takes more than 5 attempts before she makes a basket.

a. Geometric no "n" $\leftarrow$ fixed # of trials $p = .64$

b. $P(X = 4) = \text{geometpdf}(.64, 4) = .02986$

dgeom

```
> dgeom(3,.64)
[1] 0.02985984
```

c. $E[x] = \frac{1}{p} = \frac{1}{.64} = 1.5625 \approx 2$

d. $P(X > 5) = (1-p)^5 = (.36)^5 = .006$

$1 - P(X \leq 5) = 1 - \text{geometcdf}(.64, 5)$

$1 - \text{pgeom}(4, .64)$

$$\boxed{P(X > a) = (1-p)^a}$$

only for geometric

$\boxed{\text{cdf} \leq}$ $\boxed{\text{pdf} =}$

30. The probabilities that a customer selects 1, 2, 3, 4, or 5 items at a convenience store are 0.32, 0.12, 0.23, 0.18, and 0.15, respectively.

a. Construct a probability distribution for the data

X	1	2	3	4	5
P(X)	.32	.12	.23	.18	.15

b. Find $P(X > 3.5)$ $= .18 + .15 = .33$

c. Find $P(1.0 < X < 3.0)$ $= .12$

d. Find $P(X < 5)$ $= .32 + .12 + .23 + .18 = 1 - .15 = .85$

12. Of the automobiles produced at a particular plant, 40% had a certain defect.

a. What is the probability that more than 50 cars will need to be inspected before one with the defect is found?

b. What is the probability that the twentieth car inspected will have a defect?

c. Suppose a company purchases five of these cars. What is the probability that exactly one of the five cars has a defect?

Geometric $p = .4$ no fixed # of trials

a. $P(X > 50) = (1 - .4)^{50}$

$1 - P(X \leq 50)$

$1 - \text{geometcdf}(.4, 50)$

$1 - \text{pgeom}(49, .4)$

```
> 1-pgeom(49,.4)
[1]  8.082868e-12
```

≈ 0

b. $P(X = 20) = \text{geometpdf}(.4, 20)$

$\text{dgeom}(19, .4)$

R Studio

geometric

use 1 less

c. changes this to a binomial distribution $\Rightarrow n = 5$

$P(X = 1) = \text{binompdf}(5, .4, 1)$

$\text{dbinom}(1, 5, .4)$

21. What kind of variable? Categorical or quantitative? If quantitative, discrete or continuous?

a. Score on the final exam (out of 100 points) as recorded on report card. – 0, 1, 2, 3, ... , 99, 100

quant, discrete

b. Final grade for the course (A, B, C, D, F).

c. The amount a person grew (in height) in a year.

d. The number of classes a student missed.

b. categorical

c. quant, continuous

d. quant, discrete

7. Five items are selected at random from a production line. What is the probability of exactly 2 defectives if it is known that the probability of a defective item is .05?

Binomial

5 trials ($n = 5$)

$p = .05$

trials independent

$P(X=2) = \text{binompdf}(5, .05, 2) = .0214$

dbinom(2, 5, .05)

Question 13

Which statement is not true for a binomial distribution with n = 10 and p = 1/20 ? .05

a) The highest probability occurs when x equals 0.5000 $P(X=.5)$ is highest prob. not a valid trial attempt

b) The number of trials is equal to 10 ✓

c) The standard deviation is 0.6892 ✓ $\sqrt{np(1-p)} = \sqrt{10(.05)(.95)}$

d) The probability that x equals 1 is 0.3151 ✓ $P(X=1) = .3151$ binompdf(10,.05,1)

e) The mean equals 0.5000. ✓ $np = 10(1/20) = .5$

f) None of the above

Question 7

$S = \{2, 9, 12, 28, 40\}$

Let A = {2, 9}, B = {9, 12, 28}, D = {40} and S = sample space = $A \cup B \cup D$. Identify $A^c \cap B$.

$A^c = \{12, 28, 40\}$

a) {9}

b) {9, 12, 28}

c) {2, 12, 28}

d) {9, 40}

e) {12, 28}

f) None of the above.