Mean Value Theorem for integrals

$$\int_a^b f(x)dx = f(c)(b-a)$$

Average Value Theorem

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Natural Logarithmic Functions

$$\ln x = \int_1^x \frac{1}{t}dt \quad x>0$$

$$\ln(1)=0$$

$$\ln(ab)=\ln a+\ln b$$

$$\ln\left(a^n\right)=n\ln a$$

$$\ln\left(\frac{a}{b}\right)=\ln a-\ln b$$

$$\ln\left(a^{\frac{p}{q}}\right)=\frac{p}{q}\ln a$$

$$\ln e=\int_1^e \frac{1}{t}dt=1$$

$$\frac{d}{dx}[\ln u]=\frac{1}{u}\frac{du}{dx}=\frac{u'}{u}$$

$$\frac{d}{dx}[\ln|u|]=\frac{u'}{u}$$

$$\int\frac{1}{x}dx=\ln|x|+C$$

$$\int\sin u\,du=-\cos u+C$$

$$\int\cos u\,du=\sin|u|+C$$

$$\int\tan u\,du=-\ln|\cos u|+C$$

$$\int\cot u\,du=\ln|\sin u|+C$$

$$\int\sec u\,du=\ln|\sec u+\tan u|+C$$

$$\int\csc u\,du=-\ln|\csc u+\cot u|+C$$

Logarithmic Differentiation

$$g'(x)=g(x)\left(\frac{g_1'(x)}{g_1(x)}+\frac{g_2'(x)}{g_2(x)}+\ldots\ldots+\frac{g_n'(x)}{gn(x)}\right)$$

Inverse Function

$$\left(f^{-1}\right)'(b)=\frac{1}{f'(a)}=\frac{1}{f'\left(f^{-1}(b)\right)}$$

Exponential Function

$$f^{-1}(x)=e^x$$

$$y=e^x \quad iff \quad x=\ln y$$

$$\ln e^x=x$$

$$e^{\ln x}=x$$

$$e^a e^b=e^{a+b}$$

$$\frac{e^a}{e^b}=e^{a-b}$$

$$\frac{d}{dx}\left[e^u\right]=e^u\frac{du}{dx}$$

$$\int e^u du=e^u+C$$

$$\int e^{g(x)}g'(x)du=e^{g(x)}+C$$

$$a^x=e^{(\ln a)x}$$

$$C=\log_B A \quad iff \quad B^C=A$$

$$\log_a x=\frac{\ln x}{\ln a}$$

$$y=a^x \quad iff \quad x=\log_a y$$

$$a^{\log_a x}=x, \quad x>0$$

$$\log_a a^x=x$$

$$\frac{d}{dx}\left[a^u\right]=(\ln a)a^u\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u]=\frac{1}{(\ln a)u}\frac{du}{dx}$$

$$\int a^x dx=\left(\frac{1}{\ln a}\right)a^x+C$$

$$\frac{d}{dx}\left[u^n\right]=nu^{n-1}\frac{du}{dx}$$

ARBITRARY POWERS

$$x^r=e^{r\ln x}$$

$$x^{r+s}=x^r x^s=e^{(r+s)\ln x}=e^{r\ln x}e^{s\ln x}=x^{rs}$$

$$\frac{d}{dx}\left(p^u\right)=p^u\ln p\frac{du}{dx}$$

$$\int p^x dx=\frac{1}{\ln p}p^x+C$$

Exponential Growth and Decay

$$y(t)=kt+C$$

$$y(t+\Delta t)=k(t+\Delta t)+C=(kt+C)+k\Delta t$$

$$=y(t)+k\Delta t$$

$$y(t)=Ce^{kt}$$

$$y(t+\Delta t)=Ce^{k(t+\Delta t)}=Ce^{kt+k\Delta t}=Ce^{kt}\bullet e^{k\Delta t}y(t)$$

$$f(t)=Ce^{kt}$$

$$f'(t)=Ce^{kt}=kCe^{kt}=kf(t)$$

Population Growth

Growth Constant

$$P'(t)=kP(t)$$

Size of population at any Time

$$P(t)=P(0)e^{kt}$$

Radioactive Decay

Decay Constant

$$A'(t)=kA(t)$$

$$A(t)=A(0)e^{kt}$$

$$kT=-\ln 2$$

Compound Interest

Continuous Compound

$A(t)=A_0e^{rt}$ — *$A(t)$ = principal dollars at time*

A_0 = initial investment

r = annual interest rate

Inverse Trigonometric Fuctions

Definition of Inverse Functions

Function	*Domain*	*Range*
$y=\arcsin x$ *iff* $\sin y=x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\arccos x$ *iff* $\cos y=x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\arctan x$ *iff* $\tan y=x$	$-\infty<x<\infty$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$
$y=\operatorname{arccot} x$ *iff* $\cot y=x$	$-\infty<x<\infty$	$0<y<\pi$
$y=\operatorname{arcsec} x$ *iff* $\sec y=x$	$\lvert x\rvert\ge 1$	$0\le y<\pi,\ y\ne\frac{\pi}{2}$
$y=\operatorname{arccsc} x$ *iff* $\csc y=x$	$\lvert x\rvert\ge 1$	$-\frac{\pi}{2}\le y<\frac{\pi}{2},\ y\ne 0$

The Inverse Sine

$$\sin\left(\sin^{-1}x\right)=x \quad x\in[-1,1]$$

$$\sin^{-1}(\sin x)=x \quad x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$\sin^{-1}(-x)=-\sin^{-1}x$$

$$y=\sin^{-1}x, \quad domain:(-1,1), \quad range:\left(-\frac{1}{2}\pi,\frac{1}{2}\pi\right)$$

$$y=\sin x, \quad domain:\left(-\frac{1}{2}\pi,\frac{1}{2}\pi\right), \quad range:(-1,1)$$

$$\sin\left(\sin^{-1}x\right)=x \qquad \csc\left(\sin^{-1}x\right)=\frac{1}{x}$$

$$\cos\left(\sin^{-1}x\right)=\sqrt{1-x^2} \qquad \sec\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$$

$$\tan\left(\sin^{-1}x\right)=\frac{x}{\sqrt{1-x^2}} \qquad \cot\left(\sin^{-1}x\right)=\frac{\sqrt{1-x^2}}{x}$$

$$\frac{d}{dx}\left(\sin^{-1}u\right)=\frac{1}{\sqrt{1-u^2}}\bullet\frac{du}{dx}$$

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+C$$

Derivatives of Inverse Trigonometric

$$\frac{d}{dx}[\arcsin u]=\frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u]=\frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u]=\frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u]=\frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u]=\frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\operatorname{arccsc} u]=\frac{-u'}{|u|\sqrt{u^2-1}}$$

Integration of Inverse Trigonometric

$$\int\frac{1}{\sqrt{a^2-u^2}}du=\arcsin\frac{u}{a}+C$$

$$\int\frac{1}{a^2+u^2}du=\frac{1}{a}\arctan\frac{u}{a}+C$$

$$\int\frac{1}{u\sqrt{u^2-a^2}}du=\frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+C$$

Hyperbolic Functions

$$\sinh x=\frac{e^x-e^{-x}}{2} \qquad \operatorname{csch} x=\frac{1}{\sinh x},\ x\ne 0$$

$$\cosh x=\frac{e^x+e^{-x}}{2} \qquad \operatorname{sech} x=\frac{1}{\cosh x}$$

$$\tanh x=\frac{\sinh x}{\cosh x} \qquad \coth x=\frac{1}{\tanh x},\ x\ne 0$$

Hyperbolic Identities

$$\cosh^2 x-\sinh^2 x=1 \qquad \sinh(x+y)=\sinh x\cosh y+\cosh x\sinh y$$

$$\tanh^2 x+\operatorname{sech}^2 x=1 \qquad \sinh(x-y)=\sinh x\cosh y-\cosh x\sinh y$$

$$\coth^2 x-\operatorname{csch}^2 x=1 \qquad \cosh(x+y)=\cosh x\cosh y+\sinh x\sinh y$$

$$\cosh(x-y)=\cosh x\cosh y-\sinh x\sinh y$$

$$\sinh^2 x=\frac{-1+\cosh 2x}{2} \qquad \cosh^2 x=\frac{1+\cosh 2x}{2}$$

$$\sinh^2 x=2\sinh x\cosh x \qquad \cosh 2x=\cosh^2 x+\sinh^2 x$$

$$\frac{d}{dx}[\sinh u]=(\cosh u)u'$$

$$\frac{d}{dx}[\cosh u]=(\sinh u)u'$$

$$\frac{d}{dx}[\tanh u]=\left(\operatorname{sech}^2 u\right)u'$$

$$\frac{d}{dx}[\coth u]=-\left(\operatorname{csch}^2 u\right)u'$$

$$\frac{d}{dx}[\operatorname{sech} u]=-(\operatorname{sech} u\tanh u)u'$$

$$\frac{d}{dx}[\operatorname{csch} u]=-(\csc u\coth u)u'$$

$$\int\cosh u\,du=\sinh u+C$$

$$\int\sinh u\,du=\cosh u+C$$

$$\int\operatorname{sech}^2 u\,du=\tanh u+C$$

$$\int\operatorname{csch}^2 u\,du=-\coth u+C$$

$$\int\operatorname{sech} u\tanh u\,du=-\operatorname{sech}+C$$

$$\int\operatorname{csch} u\coth u\,du=-\operatorname{csch} u+C$$

$$\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$$

$$\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$$

$$\tanh^{-1}x=\frac{1}{2}\ln\frac{1+x}{1-x}$$

$$\coth^{-1}x=\frac{1}{2}\ln\frac{x+1}{x-1}$$

$$\operatorname{sech}^{-1}x=\ln\frac{1+\sqrt{1-x^2}}{x}$$

$$\operatorname{csch}^{-1}x=\ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{|x|}\right)$$

The Inverse Tangent

$$y=\tan^{-1}x,\ domain:\ (-\infty,\infty),\ range:\ \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$y=\tan\ x,\ domain:\ \left(-\frac{\pi}{2},\frac{\pi}{2}\right),\ range:\ (-\infty,\infty)$$

$$\tan\left(\tan^{-1}x\right)=x \quad for\ all\ real\ x \qquad \tan^{-1}(\tan x)=x \quad x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$\tan\left(\tan^{-1}x\right)=x \qquad \cot\left(\tan^{-1}x\right)=\frac{1}{x}$$

$$\sin\left(\tan^{-1}x\right)=\frac{x}{\sqrt{1+x^2}} \qquad \cos\left(\tan^{-1}x\right)=\frac{x}{\sqrt{1+x^2}}$$

u

$$\sec\left(\tan^{-1}x\right)=\sqrt{1+x^2} \qquad \csc\left(\tan^{-1}x\right)=\frac{\sqrt{1+x^2}}{x}$$

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+u^2}\frac{du}{dx}$$

$$\int\frac{1}{a^2+u^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

Inverse Secant

$$y=\sec^{-1}x,\ domain:\ (-\infty,1)\cup(1,\infty),\ range:\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$$

$$y=\sec x,\ domain:\ \left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right),\ range:(-\infty,1)\cup(1,\infty)$$

$$\sec\left(\sec^{-1}x\right)=x,\ |x|\ge 1$$

$$\sec^{-1}(\sec x)=x, \quad x\in\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$$

$$\sec\left(\sec^{-1}x\right)=x \qquad \csc\left(\sec^{-1}x\right)=\frac{x}{\sqrt{x^2-1}}$$

$$\sin\left(\sec^{-1}x\right)=\frac{\sqrt{x^2-1}}{x} \qquad \cos\left(\sec^{-1}x\right)=\frac{1}{x}$$

$$\tan\left(\sec^{-1}x\right)=\sqrt{x^2-1} \qquad \cot\left(\sec^{-1}x\right)=\frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}\left(\sec^{-1}u\right)=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\int\frac{1}{x\sqrt{x^2-a^2}}dx=\frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right)+C$$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}=-\frac{d}{dx}\left(\sin^{-1}x\right)$$

$$\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}=-\frac{d}{dx}\left(\tan^{-1}x\right)$$

$$\frac{d}{dx}\left(\csc^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}=-\frac{d}{dx}\left(\sec^{-1}x\right)$$

$$\frac{d}{dx}\left[\sinh^{-1}u\right]=\frac{u'}{\sqrt{u^2+1}}$$

$$\frac{d}{dx}\left[\cosh^{-1}u\right]=\frac{u'}{\sqrt{u^2-1}}$$

$$\frac{d}{dx}\left[\tanh^{-1}u\right]=\frac{u'}{1-u^2}$$

$$\frac{d}{dx}\left[\csc^{-1}u\right]=\frac{-u'}{|u|\sqrt{1+u^2}}$$

$$\frac{d}{dx}\left[\operatorname{csch}^{-1}u\right]=\frac{u'}{u\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left[\cot^{-1}u\right]=\frac{u'}{1-u^2}$$

$$\int\frac{1}{\sqrt{u^2\pm a^2}}du=\ln\left(u+\sqrt{u^2\pm a^2}\right)+C$$

$$\int\frac{1}{\sqrt{a^2-u^2}}du=\frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right|+C$$

$$\int\frac{1}{u\sqrt{a^2\pm u^2}}du=-\frac{1}{a}\ln\frac{a+\sqrt{a^2\pm u^2}}{|u|}+C$$

$$\boxed{f'(x_0)(X-X_0)+f(x_0)=y}$$