A Bunch of Series Examples:

1. Geometric: $\sum_{n=0}^{\infty} \frac{2^{n+2}}{3^n}$

$$\sum_{n=0}^{\infty} \frac{2^{n+2}}{3^n} = \sum_{n=0}^{\infty} \frac{2^2 \cdot 2^n}{3^n} = 4\sum_{n=0}^{\infty} \frac{2^n}{3^n} = 4\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n \quad |r|<1 \quad S = 4\left(\frac{1}{1-2/3}\right) = 12$$

2. Using Partial Fractions and Geometric: $\sum_{n=0}^{\infty} \left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right]$

$$\sum_{n=0}^{\infty} \left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right] = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n - \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{1}{1-2/3} - \left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots\right] = 3-1 = 2$$

3. Basic Divergence Test

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$$

$$\lim_{n->\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0 => diverges \ (thm\ 11.6.1)$$

4. Basic Divergence Test

$$\sum_{n=1}^{\infty} \frac{n!}{2^n}$$

$$\lim_{n->\infty} \frac{n!}{2^n} = \infty \neq 0 => diverges \ (thm\ 11.6.1)$$

5. Geometric

$$\sum_{n=0}^{\infty} 2\left(-\frac{1}{2}\right)^n \Rightarrow \textit{Geometric with } |r| = \left|-\frac{1}{2}\right| < 1 \Rightarrow \textit{converges}$$

$$\textit{converges to } 2\left(\frac{1}{1-\left(-\frac{1}{2}\right)}\right) = \frac{4}{3}$$

6. Partial Fractions

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \sum_{n=1}^{\infty}\left(\frac{1}{2n} - \frac{1}{2(n+2)}\right) \textit{ partial fractions}$$

$$= \left(\frac{1}{2}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{8}\right)+\left(\frac{1}{6}-\frac{1}{10}\right)+\left(\frac{1}{8}-\frac{1}{12}\right)+\left(\frac{1}{10}-\frac{1}{14}\right)+\ldots = \frac{1}{2}+\frac{1}{4} = \frac{3}{4}$$

7. Geometric

$$\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n \textit{ Geometric series}$$

$$|r| = \left|-\frac{1}{2}\right| < 1 \Rightarrow \textit{converges}$$

$$\textit{it converges to } \frac{1}{1-(-.5)} = \frac{2}{3}$$

8. Geometric

$$\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - \frac{1}{3^n}\right) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n \textit{ Geometric series}$$

$$|r| < 1 \Rightarrow \textit{converges}$$

$$\textit{it converges to } \frac{1}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{3}} = \frac{1}{2}$$

9. Basic Divergence Test

$$\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$$

$$\lim_{n\to\infty} \frac{n+1}{2n-1} = \frac{1}{2} \neq 0 \Rightarrow \textit{diverges}$$

10. Basic Divergence Test

$$\sum_{n=1}^{\infty} \frac{n}{\ln n}$$

$$\lim_{n->\infty} \frac{n}{\ln n} = \lim_{n->\infty} \frac{1}{1/n} = \lim_{n->\infty} n = \infty \neq 0 => \textit{diverges}$$

11. Geometric

$$1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + ... = \sum_{n=0}^{\infty} \left(-\frac{x}{2} \right)^n = \frac{1}{1 + x/2} = \frac{2}{2 + x}$$

$$\textit{geometric series } r = -\frac{x}{2} => x < |2|$$

12. Geometric

$$\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + ... = \sum_{n=0}^{\infty} \frac{1}{r} \left(\frac{1}{r} \right)^n = \frac{1}{r} * \frac{1}{1 - 1/r} = \frac{1}{r - 1}$$

$$\textit{geometric series} => \textit{if } \left| \frac{1}{r} \right| < 1 \textit{ then } |r| > 1$$

13. Integral Test

$$\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$$

$$\textit{let } f(x) = \frac{1}{x^{1/3}}; f \textit{ positive, decreasing, continuous for } x \geq 1 => \textit{can use Integral test}$$

$$\int_1^{\infty} \frac{1}{x^{1/3}} dx = \left[\frac{3}{2} x^{2/3} \right]_1^{\infty} = \infty => \textit{diverges}$$

14. P Series

$$\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$$

$$\textit{convergent } p-\textit{series with } p = \frac{4}{3} > 1$$

15. P Series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\textit{convergent } p-\textit{series with } p = 2 > 1$$

16. P Series

$$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$$

$divergent\ \ p-series\ with\ p=\frac{2}{3}<1$

17. Integral Test

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$$

$let\ \ f(x)=\frac{1}{x\sqrt{x^2-1}};f\ \ positive, decrea\sin g, continuous\ \ for\ \ x\geq 2 \Rightarrow can\ use\ Integral\ test$

$$\int_2^{\infty} \frac{1}{x\sqrt{x^2-1}}dx=\left[arc\sec x\right]_2^{\infty}=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6} \Rightarrow converges$$

18. Geometric

$$\sum_{n=0}^{\infty} (1.075)^n$$

$Geometric\ series\ with\ |\,r\,|>1 \Rightarrow divergent$

19. P Series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n^2}-\frac{1}{n^3}\right)=\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)-\sum_{n=1}^{\infty}\left(\frac{1}{n^3}\right)$$

$both\ convergent\ \ p-series\ with\ \ p=2>1\ \&\ \ p=3>1$

Some questions from a really old test I gave:

True/False

__F__1. For a p-series to converge, p must be less than or equal to 1.

__F__2. In the application of the Integral test, the sum is equal to the value of the integral.

__T__4. If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

__F__5. If $0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Determine the convergence or divergence for each sequence with the given general term. Choose the test used from:

(A) N^{th} term test (B) Integral test (C) Geometric Series (D) P-Series

(E) Telescoping (Partial fractions) (F) Direct Comparison (G) Limit Comparison

	Series	Test used	Converge or Diverge?	Work:
6.	$\sum_{n=4}^{\infty} \frac{1}{3n^2 - 2n - 15}$	G	Converges	Compare to 1/n^2
7.	$\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$	B	Converges	
8.	$\sum_{n=1}^{\infty} \frac{n}{2n+3}$	A	Diverges	
9.	$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$	F	Converges	
10.	$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$	F	Converges	
11.	$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$	D	Diverges	p-series p<1
12.	$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$	E	Converges	
13.	$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2}\right)^n$	C	Converges	
14.	$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$	G	Diverges	Compared with 1/n