Math 1431 Online, Midterm Review

Fall 2008

1. Find the following limits (if they exist):

   a. $\lim_{x->0} \frac{\sin 4x}{5x} =$

   b. $\lim_{x->0} \frac{\sqrt{4+x}-2}{x} =$

   c. $\lim_{x->0} \frac{\left(\frac{1}{x+1}-1\right)}{x} =$

   d. $\lim_{x->-3} \frac{x^2+x-6}{x^2-9} =$

   e. $\lim_{x->0} \left(x\left(2-\frac{1}{x}\right)\right) =$

   f. $\lim_{x->0} \frac{2\sin x \cos x}{2x} =$

   g. $\lim_{x->0} \frac{5x}{\tan(9x)} =$

   h. $\lim_{x->0} \frac{\sin(x^2)}{6x} =$

2. Determine if the following are continuous. If the function is not continuous, state the type of discontinuity.

   a. $f(x) = \begin{cases} x^2+1 & x<1 \\ 8 & x=1 \\ x^3 & x>1 \end{cases}$

   b. $f(x) = \begin{cases} 2x^2 & x<2 \\ 8 & x=2 \\ x^3 & x>2 \end{cases}$

   c. $f(x) = \begin{cases} 5-x & x<-2 \\ 7 & x=-2 \\ x^2-5 & x>-2 \end{cases}$

   d. $f(x) = \begin{cases} x^2 & \text{if } x<-1 \\ x+2 & \text{if } x\geq -1 \end{cases}$

3. Let $f(x)=\begin{cases} \dfrac{x^2-1}{x-1} & if\ x\neq 1 \\ 4 & if\ x=1 \end{cases}$

Which of the following statements, I, II and III are true?

I. $\lim\limits_{x->1} f(x)\ exists$ II. $f(1)$ exists III. f is continuous at x = 1

4. Let $f(x)=\begin{cases} x+1 & if\ x\neq 3 \\ k & if\ x=3 \end{cases}$

For what value of *k* would *f(x)* be continuous at *x=3*?

5. Find A and B so that *f*(x) is continuous:

$$f(x)=\begin{cases} 6x^2-1 & x<-1 \\ A & x=-1 \\ Bx+3 & x>-1 \end{cases}$$

6. Given $f(x)=\begin{cases} 2x & x>3 \\ x^2-3 & x\leq 3 \end{cases}$, find $f'(3)$ if it exists.

7. Find the derivative of the following:

   a. $f(x)=3(2x-1)^4$
   b. $y=\sec^3(2x)$
   c. $f(x)=3\sqrt{x}+\dfrac{5}{x}$
   d. $f(x)=\dfrac{1}{\sqrt{x}}$
   e. $f(x)=\dfrac{\sqrt{x}+2x}{x^2}$
   f. $f(x)=(x^2+2x)^4(x-1)^3$
   g. $f(x)=\dfrac{(x-1)(x+1)}{x+2}$
   h. $y=x\sqrt{(x^3+5x)}$
   i. $f(x)=\dfrac{1+\cos x}{1-\cos x}$
   j. $f(x)=\sin^4(4x^2-6x+1)$

k. $y = \frac{\cot x}{x^2}$

l. $f(\theta) = \sec\theta - \tan\theta$

8. Find $\frac{dy}{dx}$ using implicit differentiation.

   a. $x^2 + y^2 - 4x + 3y = 7$
   b. $\sin x - \cos y - 2 = 0$
   c. $x^3 - xy + y^3 = 1$
   d. $y\sqrt{x} - x\sqrt{y} = 16$
   e. $xy = 10$
   f. $x\sin 2y = 1$
   g. $x^{2/3} + y^{2/3} = 5$
   h. $\cos(x + y) = 4xy$

9. Use differentials to approximate:
   a. $\sqrt[3]{8.01}$
   b. $\cos(31^\circ)$

10. Use the definition of derivative to find the derivative of $f(x) = 3x^2 - x + 2$

11. Find $\frac{d^3}{dx^3}\left(\frac{3}{4}x^4 - 2x^3 + x - 10\right)$

12. Find $\frac{dy}{dx}$ at $x = -2$ for $y = (4x+1)(1-x)^3$

13. Find the second derivative at the point (-2,1) for $x^2 - y^2 = 3$

14. Use interval notation to give the solution set to:
    a. $x(2x-1)(3x+4) \le 0$
    b. $x^2 - 7x + 6 > 0$

15. Find: $\frac{d}{dx}\left((2x-5)\left(\frac{d}{dx}(2x^2 + x)\right)\right)$

16. A particle is moving along the parabola $y^2 = 4(x+2)$. As it passes through the point (7, 6), its y-coordinate is increasing at the rate of 3 units per second. How fast is the x-coordinate changing at this instant?

17. A man is standing on the top of a 15 foot ladder, which is leaning against a wall. Some scientific minded joker comes up and starts to pull the bottom of the ladder away at a steady rate of 6 ft/min. At what rate is the man on the ladder descending if he remains standing on the top rung when the bottom of the ladder is 9 ft. from the wall?

18. A point moves along the curve $y = 2x^2 + 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = \frac{3}{2}$?

19. On a morning of a day when the sun will pass directly overhead, the shadow of an 40-ft building on level ground is 30 feet long. At the moment in question, the angle the sun makes with the ground is increasing at the rate of $\pi/1500$ radians/minute. At what rate is the shadow length decreasing?

20. Write the equation of the tangent and normal line to
    a. $y^2 - x + 6 = 0$ at the point $(15,3)$.
    b. $2x^2 - 6xy + y^2 = 9$ at the point $(1,-1)$.

21. Find the critical numbers of *f* and classify the extreme values given $x \in [-3,1]$

$$f(x) = \frac{x}{x^2 + 4}$$

22. Find all local extreme and intervals of increasing and decreasing for the following functions:
    a. $f(x) = \frac{4}{3}x^3 - 6x + 2$
    b. $f(x) = \sqrt{3}\sin x + \cos x \quad for\ 0 \le x \le 2\pi$