Section 1.1 The Coordinate Plane

Definition A point P in the coordinate plane is located by a unique **ordered pair** of numbers *(x,y)*. The number x is called the **abscissa** (or **x-coordinate**), and the number y is called the **ordinate** (or **y-coordinate**).

Examples: Locate the following points in the xy-plane: 1. $A(2,1)$; 2. $B(-3,1)$; 3.$C(-2,-4)$, 4. $D(2,-3)$, 5. $E(-3,0)$, 6. $F(0,-1)$.

Quadrants

The two axis x and y divide the xy-plane into four regions (or quadrants): Quadrant I is composed of all the points (x,y) such that $x > 0$ and $y > 0$, then move counterclockwise to get to quadrant II, where the points (x,y) are such that $x < 0$ and $y > 0$, quadrant III where the points (x,y) are such that $x < 0$ and $y < 0$, quadrant IV wherethe points (x,y) are such that $x > 0$ and $y < 0$.

Distance Formula and Midpoint formula • Let $A(x_1, y_1)$, and $B(x_2, y_2)$ represent any two distincts points in the xy-plane. The **Euclidean distance** between the points A and B can be computed using the formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
• The coordinates of the **midpoint** of the points A and B described above are given by the formula $M\left(\frac{x_1 + x_2}{2}), \frac{y_1 + y_2}{2}\right)$

Examples 1. Let $A(3,-2)$, $B(-1,3)$. Find the distance between points A and B, as well as the coordinates of the midpoint M.

Section 1.2 Lines: Slopes, Equations, Intercepts, Parallel and Perpendicular Lines

Slope of a Line The **Slope** m of a line that passes through two points $A(x_1, y_1)$, and $B(x_2, y_2)$ is given by
$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$, assuming that $x_1 \neq x_2$. If $x_1 = x_2$, then the line is vertical (equation: $x = x_1$), and its slope is undefined.

Examples: Find the slope of the line that passes through the given points: 1. $A(-2,4)$ and $B(-1,6)$; 2. $A(-2,4)$ and $B(-2,7)$; 3. $A(-3,4)$ and $B(0,4)$.

The Equation of a Line

• The **point-slope form of the equation of a line** is given by $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope of the line.
• The **slope-intercept form of the equation of a line** is given by $y = m\,x + b$, where m is the slope of the line and b is the y-intercept.

Examples 1. Find an equation of the line that passes through the point $(2,-1)$ and has slope $m = -\frac{2}{3}$. Write the final answer in the format $y = mx + b$, and in the format $Ax + By = C$, and graph the line. 2. repeat 1. for the points $(-3,-1)$, and $(2,5)$. 3. Find an equation of the line with slope $\frac{2}{3}$, and y-intercept -1. Sketch the line. 4. Find the slope and the y-intercept of the line $3x + 7y = -1$.

Intercepts of Lines

A line with equation $Ax + By = C$, where $A \neq 0$, and $B \neq 0$, will intersect both

the x-axis and the y-axis. The **x-intercept** is the point where the line intercepts the x-axis, and its x-coordinate is found by setting y equal to zero in the equation of the line, and solving the result for x.
Similarly, the /bf y-intercept is the point where the line intercepts the y-axis, and its y-coordinate is found by setting x equal to zero in the equation of the line, and solving the result for y.
Examples Find the x- and y intercepts of the following lines: 1. $-x+2y=2$, 2. $-2x+5y=4$

Parallel and Perpendicular Lines
Two (non-vertical) lines with slopes m_1, and m_2 are **parallel** if and only if their slopes are equal.
Two (non-vertical) lines with slopes m_1, and m_2 are **perpendicular** if and only if the product of their slopes is equal to -1.

Examples 1. Find an equation of the line that passes through the point $(2,3)$ and is parallel to the line $3x-2y=5$. 2. Find an equation of the line that passes through the point $(2,-1)$ and is perpendicular to the line $4x+y=2$.

Section 1.3 Graphs of Equations
Definition The **graph** of an equation is the set of all points that satisfy the given equation. A point (x,y) will satisfy an equation in x and y if the equation is true when the coordinates of the point are substituted into the equation.
One method of graphing an equation is to build a table of method.
Examples: Graph: 1. $y=2-x$; 2. $y=x^2+2$; 3. $y=|x+1|$.
Intercepts of Graphs
An x-intercept of a graph is the x-coordinate of a point where the graph intersects the x-axis. To find the x-intercepts of a graph, set the other variable $y=0$ and solve the resulting equation for x.
A y-intercept of a graph is the y-coordinate of a point where the graph intersects the y-axis. To find the y-intercepts of a graph, set the other variable $x=0$ and solve the resulting equation for y.
Examples: Find the x and y intercepts of the equations: $x=y^2-2$, $y=x^2+1$, $y=2x+3$.
Symmetry • If the point $(x,-y)$ is on the graph of an equation whenever the point (x,y) is on the graph, then the graph is **symmetric with respect to the x-axis**
• If the point $(-x,y)$ is on the graph of an equation whenever the point (x,y) is on the graph, then the graph is **symmetric with respect to the y-axis**
• If the point $(-x,-y)$ is on the graph of an equation whenever the point (x,y) is on the graph, then the graph is **symmetric with respect to the origin**
Ex: Study the symmetries of $y=x^2+1$, $y=x^3$, $x^2+y^2=1$, $y=|x-1|$, $x=-y^2$
Circles

Definition: A **circle** is the set of all points that are at a fixed distance r from a fixed point $C(h,k)$. The point $C(h,k)$ is called the center of the circle. The equation of a circle in standard form is given by:
$(x-h)^2+(y-k)^2=r^2$.
Examples:
- Show that
$x^2+y^2-6x+10y-2=0$ represents the equation of a circle.
- Find the equation of the circle that satisfies the condition that the endpoints of a diameter are (-6,7) and (4,5).

Section 2.1 Algebraic and Graphical Solutions of Equations
To solve an equation in the variable x using the algebraic method is to use the rules of algebra to isolate the unknown x on one side of the equation.
To solve an equation in the variable x in using the graphical method is to move all terms to one side of the equation and set those terms equal to y. Sketch the graph to find the value of x where $y=0$.
Examples $-x-1=2x+2$; $6x+3=4x+33$; $2(x-3)+7=-4(x+1)+3$; $2x+\frac{x}{12}-\frac{x-3}{6}=x$; $\frac{1}{2}x+1=3x$ (graphical solution).

Section 6.1 Two by two linear systems
To solve a system of two linear equations $\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$ means to find values for x and y that satisfy both equations.
The system will have exactly one solution, or infinitely many (consistent systems), or no solution (inconsistent systems.)
Examples: $\begin{cases} 3x-y=-2 \\ 6x-2y=3 \end{cases}$ $\begin{cases} x+y=4 \\ 3x+2y=7 \end{cases}$ $\begin{cases} 4x+y=8 \\ 8x+2y=-4 \end{cases}$ $\begin{cases} 3x+y=5 \\ x-2y=4 \end{cases}$ $\begin{cases} 4x+y=9 \\ 3x-2y=-7 \end{cases}$
$\begin{cases} 3x+y=-1 \\ 15x+5y=-2 \end{cases}$ $\begin{cases} x-2y=4 \\ -3x+6y=-12 \end{cases}$

Section 2.2 Modeling with Equations
Using Modeling to Solve Problems - Guidelines
Step 1: Identify the variable.
Step 2: Express all unknown quantities in terms of one variable.
Step 3: Set up the equation.
Step 4: Solve the equation and check your results.
Ex. 1: Irving paints on paper that is 30 in. by 12 in. high. He then places his painting on a mat so that a uniformly wide strip of mat shows all around the picture. The perimeter of the mat is 120 in. How wide is the strip of mat showing around the picture?
Ex. 2: Consecutive Integer Problem: The sum of three consecutive integers is 705. What is the smallest integer?
Ex 3. Randy and Alice, together, have collected 143 DVD's. If Randy has ten

fewer than half the number of DVD's than those in Alice's collection, how many DVd's has Alice collected?
Ex. 4: One half of Janice's age plus one-third her age 3 years from now is 36 years. how old is Janice?
Ex. 5: Ed invests $\$6,000$ at 5% per year. How much additional money must he invest at 8% per year so that the total interest he earns on the two investments after one year is $\$1,260$?
Ex. 6: Chris and Mary live 13 miles apart on the same road. They leave their homes at the same time and begin cycling toward each other. If Chris cycles at the rate of 14 miles per hour and Mary at the rate of 12 miles per hour, how long will it take for them to meet?
Ex. 7: There are twice as many nickels as quarters in a piggy bank. If the value of these coins is $\$29.75$, how many quarters are in the piggy bank?

Section 2.3 solving quadratic Equations

A quadratic equation is an equation that can be written in the form $ax^2+bx+c=0$ where $a, b,$and c are real #s with $a \neq 0$.

Solving Quadratic Equations by Factoring

To solve a quadratic equation by factoring, rewrite the equation, if necessary, so that one side is equal to 0 and use the **Zero-Product Property:**
$ab = 0$ if and only if $a = 0$ or $b = 0$.
Ex1: $2x^2 + 7x + 3 = 0$; Ex2: $x^2 = 5(x + 100)$; Ex3: $x^2 + 8x = -12$; Ex4: $5x^2 = -14x + 3$; Ex5: $4x^2 = 81$; Ex.6: $6x^2 - x = 4x$; Ex7: The lenght of a rectangle is 3 feet longer than its width. If the area of the rectangle is 40 square feet, what is its width?

Solving Quadratic Equations by Completing the Square

To solve a quadratic equation $x^2 + bx + c = 0$ by the method of **completing the square**, rewrite the equation as $ax^2 + bx = -c$ and make the left-hand side a perfect square.

To make $x^2 + bx$ a perfect square, add $\left(\frac{b}{2}\right)^2$, the square of half the coefficient of x, to get $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$
To solve a quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$ and $a \neq 1$) by the method of **completing the square**, rewrite the equation as $ax^2 + bx = -c$ and then divide both sides by a to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$. Next make $x^2 + \frac{b}{a}x$ a perfect square by adding $\left(\frac{b}{2a}\right)^2$, the square of half the coefficient of x.
Ex1: $x^2 - 10x + 3 = 0$; Ex2: $2x^2 + 8x + 1 = 0$; Ex3: $x^2 - 4x - 5 = 0$; Ex4: $x^2 + 10x + 13 = 0$;Ex5: $x^2 = 1 - \frac{2}{3}x$; Ex6: $2x^2 + 8x + 3 = 0$;

Solving by the Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$ where $a \neq 0$, can be found by using the **quadratic formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex1: $5x^2 - 2x - 2 = 0$; Ex2: $4x^2 - x = 3$;Ex3: $x^2 - 4x = 5$; Ex4: $x^2 + 10x + 13 = 0$; Ex5: $2x^2 + 8x + 3 = 0$; Ex6: The sum of the squares of two positive consecutive even integers is 1460. What is the smaller integer?

The Discriminant The **discriminant** of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is given by $D = b^2 - 4ac$

If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two distinct real solutions.

If $D = 0$, then the equation $ax^2 + bx + c = 0$ has exactly one real solution.

If $D < 0$, then the equation $ax^2 + bx + c = 0$ has no real solutions.

Ex1: $x^2 - x - 6 = 0$; Ex2: $4x^2 + 12x + 9 = 0$; Ex3: $x^2 + x + 5 = 0$; Ex4: $4x^2 = 5 - 3x$; Ex5: $x^2 + 6x = -13$; Ex5: $25x^2 + 60x = -36$

Section 2.4 Complex Numbers

Definition: A **complex number** is a number that can be written in the form $a + b\,i$, where a and b are real numbers and $i^2 = -1$.

The number a is called the **real part** of the complex number $a + b\,i$ and the number b is called the **imaginary part**

Examples -3; $\frac{1}{2}i$; $6i - \sqrt{2}$. $\dfrac{-3+5i}{6}$;$\dfrac{3+\sqrt{-4}}{3}$

Addition, Substraction, and Multiplication

Addition:

$(a + bi) + (c + di) = (a + c) + (b + d)i$

Subtraction:

$(a + bi) - (c + di) = (a - c) + (b - d)i$

Multiplication:

$(a + bi)\,(c + di) = (ac - bd) + (ad + bc)\,i$

Examples:

$(2 + 3i) + (-4 + 7i)$; $(-2 - 5i) + (4 - i)$; $(-1 + 4i) - (4 - i)$. $(-3 + 4i)\,(2 - i)$;i^{102}

Division

The complex conjugate of the complex number $a + bi$ is the complex number $a - bi$.

To simplify the quotient $\dfrac{a+bi}{c+di}$, multiply both the numerator and denominator by the complex conjugate of the denominator:

$$\frac{a+bi}{c+di} = \left(\frac{a+bi}{c+di}\right)\left(\frac{c-di}{c-di}\right)$$
$$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

Examples: $\dfrac{-2-i}{3+4i}$; $\dfrac{2-3i}{i}$

Complex Roots of Quadratic Equations:
If the discriminant $D = b^2 - 4ac$ of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is negative, the equation has no real roots.
The roots of the equation are complex numbers and appear as a complex conjugate pair.
Examples: $x^2 + 36 = 0$; $x^2 + x + 1 = 0$; $4x^2 + x + 5 = 0$
Section 2.6 Solving Linear Inequalities
An inequality in the variable x is **linear** if each term is a constant or multiple of x. The inequality will contain an inequality symbol:

$<$	is less than
$\leq$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to

To solve an inequality containing a variable, find all values of the variable that makes the inequality true.

Rules for Inequalities:
1. $A \leq B$ is equivalent to $A + C \leq B + C$
2. $A \leq B$ is equivalent to $A - C \leq B - C$
3.If $C > 0$ then $A \leq B$ is equivalent to $C\,A \leq C\,B$
4. If $C < 0$ then $A \leq B$ is equivalent to $C\,A \geq C\,B$
Ex1: $2x - 5 > 3$;
Ex2: $3x + 11 \geq 6x + 8$;
Ex3: $-1 < 2x - 5 \leq 3$;
Ex4: Let $\{-4, 0, 3\}$.
What element of S, if any, satisfy the inequality $3x\ +\ 5 > x$?
Section 2.8 - Solving Absolute Value Equations
To solve an equation involving **absolute value**, use the following property:
If C is positive, then $|x| = C$ is equivalent to $x = \pm C$.

Special Cases for $|x| = C$:
Case 1: If C is negative, then the equation $|x| = C$ has no solution since absolute value cannot be negative.
Case 2: The solution of the equation $|x| = 0$ is $x = 0$. Examples: **1.** $|4x + 7| = 9$
2. $3|x + 5| + 6 = 15$
3. $|2x + 5| + 6 = 1$
4. $|4x + 1| = 0$.
5. $|x - 1| = |3x + 2|$.
6. $|3x + 2| = 7$
7. $3|x + 1| - 4 = 5$
8. $|2x + 1| + 3 = 1$

9. $|3x-5|=0$
10. $|x-4|=|2x+3|$

Solving an inequality involving **absolute value,** use the following properties:
If C is positive, then:
a. $|x|<C$ in equivalent form is
$-C<x<C$.
b. $|x|\leq C$ in equivalent form is
$-C\leq x\leq C$.
c. $|x|>C$ in equivalent form is
$x<-C$. or $x>C$.
d. $|x|\geq C$ in equivalent form is
$x\leq -C$. or $x\leq C$.
Special cases:
Case 1: If C is negative, then:
a. The inequality $|x|<C$ has no solution.
b. The inequality $|x|\leq C$ has no solution.
c. Every real number satisfies $|x|>C$.
d. Every real number satisfies $|x|\geq C$.
Case 2. a. The inequality $|x|<0$ has no solution.
b. The inequality $|x|\leq 0$ is $x=0$.
c. The inequality $|x|>0$. in equivalent form is $x<0$ or $x>0$.
d. Every real number satisfies the inequality $|x|\geq 0$.

Examples 1. Solve $\left|\frac{x-2}{3}\right|<2$.
Write the solution using interval notation and graph the solution set.
2. $|3x-1|\geq 2$. **3.** $|x+3|\leq 0$.

4. $|2x+1|>0$. **5.** $|2x-7|<3$.

6. $\left|\frac{x+3}{4}\right|>6$.

7. $|7x+3|\geq 17$. **8.** $|8x-2|>0$.

Section 3.1 Definition of a Function

A **function** is a rule that assigns to each element x is a set A , called the **domain** of f, exactly one element, called $f(x)$, in a set B. The elements in sets A and B are reals.
The symbol $f(x)$ is read "f **of** x" and is the value of f at x. The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain A.
To **evaluate** f at a number, substitute the number for x in the definition of f.
Examples **1.** Evaluate the function $f(x)=2x^2+3x-4$ at the numbers $-2, -\frac{1}{2}, 2$, and $\sqrt{2}$.

2. $g(x) = \dfrac{1-x}{1+x}$; Evaluate
$g(2), g(-2), g(a), and\ g(a+1)$.
3. Evaluate the function at -1,1, and 2.
$$f(x) = \begin{cases} x^2 - 1 & if \quad x < 2 \\ 3x + 5 & if \quad x \geq 2 \end{cases}$$
4. Express the rule "multiply by 5, subtract 12, then square" in function notation.
5. Evaluate the function $f(x) = 5x - 2$ at the numbers $-2, 0, 3,\ and\ \frac{1}{a-1}$.
6. For the function $f(x) = (x-1)^2 - 3$, evaluate $f(-1), f(4), f\left(\frac{1}{2}\right),\ and\ f(a)$.
7. For the function
$$f(x) = \begin{cases} -3x^2 & if \quad x < 0 \\ 6x + 1 & if \quad x \geq 0 \end{cases}$$
Evaluate $f(-4), f(-2), f(0),\ and\ f(6)$
Domain of a Function If the domain of a function has not been explicitly stated, then by convention the domain is the set of all real numbers for which the expression is defined as a real number. **Examples** Find the domains of the following functions
1. $f(x) = \frac{2}{x^2+5x+6}$ **2.** $f(x) = \sqrt{16 - x^2}$. **3.** $f(x) = \frac{x}{\sqrt{x+2}}$. **4.** $f(x) = \sqrt{8x - 2}$. **5.** $f(x) = \sqrt{4 - 3x}$. **6.** $f(x) = \frac{1}{4-9x}$. **7.** $f(x) = \frac{x}{x^2-8x+7}$

Section 3.2 Graphing a Function

Let f be a function with domain A. The **graph** of the function f is given by
$\{(x, y)|x \in A.\}$
The graph is the set of all points (x, y) in the coordinate plane whose x-coordinates are elements of the domain A and whose corresponding y-coordinates are given by $y = f(x)$.
The Vertical Line Test: A curve in the xy-plane is the graph of a function if and only if no vertical line intersects that curve more than once.
Examples on graphing functions:
1. $f(x) = \sqrt{x}$ **2.** $f(x) = x^2 + 1$
3. $f(x) = \begin{cases} 2 & if \quad x \leq -1 \\ x^2 & if \quad x > -1 \end{cases}$
4. $f(x) = |x| + 2$
5. $f(x) = 2x + 3$
6. $f(x) = -x^2 + 2$
7. $f(x) = \sqrt{x + 2}$
8. $f(x) = \begin{cases} 3 & if \quad x < 0 \\ x^2 + 1 & if \quad x \geq 0 \end{cases}$

Section 3.3 Direct Variation, Inverse Variation, and Joint Variation

Direct Variation: P is directly proportional to Q is expressed by the equation $P = kQ$. where k is a non-zero constant. We also say that P is proportional to Q

or P varies directly as Q.
Inverse Variation: P is inversely proportional to Q is expressed by the equation $P = \frac{k}{Q}$, where k is a nonzero constant. We also say that P varies inversely as Q.
Joint Variation: P is jointly proportional to Q and R is expressed by the equation $P = kQR$, where k is a non-zero constant.
We also say that P varies jointly as Q and R.
The constant k is called the **constant of proportionality.**
Examples: **1.** M is inversely proportional to N. (a) Write a formula to express the given statement. (b) If $N = 6$, then $M = 2$. Use the formula in part (a) and the given information to find the constant of proportionality.
2. S is directly proportional to the square of T. (a) Write a formula to express the given statement. (b) If $T = 8$, then $S = 14$. Use the formula in part (a) and the given information to find the constant of proportionality.
3. A is jointly proportional to B and C. (a) Write a formula to express the given statement. (b) If $B = 10$ and $C = 9$, then $A = 34$. Use the formula in part (a) and the given information to find the constant of proportionality.
4. M is directly proportional to B and inversely proportional to C. (a) Write a formula to express the given statement.
5. S is inversely proportional to x. (a) Write a formula to express the given statement. (b) If $N = 6$, then $M = 2$. Use the formula in part (a) and the given information to find the constant of proportionality.
6. P is inversely proportional to M. (a) Write a formula to express the given statement. (b) If $M = 6$, then $P = 24$. Use the formula in part (a) and the given information to find the constant of proportionality.
7. In a certain area, the property tax on a house is directly proportional to its assessed value. A house that is assessed at $\$125,000$ has a property tax of $\$2,950$. Find the property tax on a house that has an assessed value of $\$175,000$. **8.** xR is jointly proportional to S and T. (a) Write a formula to express the given statement. (b) If $S = 4$, and $T = 6$, then $R = 21$.
Use the formula in part (a) and the given information to find the constant of proportionality.
9. The cost of framing a poster is jointly proportional to its length and width. If a poster that is 3 feet long and 2 feet wide costs \$90 to frame, what is the cost of framing a poster that is 4 feet by 2.5 feet?

Section 3.4 Horizontal and Vertical Shifts of Graphs

Vertical Shifts: To graph the function $y = f(x) + c$ $(c > 0)$, shift the graph of $y = f(x)$ upward c units. To graph the function $y = f(x) - c$ $(c > 0)$, shift the graph of $y = f(x)$ downward c units.
Horizontal Shifts: To graph the function $y = f(x - c)$ $(c > 0)$, shift the graph of $y = f(x)$ to the right c units. To graph the function $y = f(x + c)$ $(c > 0)$, shift the graph of $y = f(x)$ to the left c units.

Examples:
1. $f(x) = x^2$ **2.** $f(x) = |x|$ **3.** $f(x) = \sqrt{x}$
Reflecting Graphs: To graph the function $y = -f(x)$, reflect the fraph of $y = f(x)$ in the x-axis. To graph the function $y = f(-x)$, reflect the graph of $y = f(x)$ in the y-axis.
Vertical Stretching and Shrinking of Graphs: To graph the function $y = a\,f(x)$ $(a > 1)$, stretch the graph of $y = f(x)$ vertically by a factor of a. Tp graph the function $y = a\,f(x)$ $(0 < a < 1)$, shrink the graph of $y = f(x)$ vertically by a factor of a.
Even and Odd Functions
Even Function: A function f is even if
$f(-x) = f(x)$ for all x in the domain of f. The graph of an even function is symmetric with respect to the y-axis.
Odd function: A function f is odd if
$f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric with respect to the origin.

Section 3.5 Quadratic Functions

A **quadratic function** f is a function that can be put in the form $f(x) = a\,x^2 + b\,x + c$, where a, b, and c are real numbers and a is non-zero.
A quadratic function f can be expressed in the form $f(x) = a(x - h)^2 + k$, called the standard form, by the method of completing the square. The graph of f is a parabola with vertex (h, k).
If $a > 0$, the parabola opens upwards and the vertex is the lowest point on the graph. The **minimum value** of f is $f(h) = k$.
If $a < 0$, the parabola opens downwards and the vertex is the highest point on the graph. The **maximum value** of f is $f(h) = k$.
Examples on graphing functions:
1. $f(x) = x^2 + 2x + 3$ **2.** $f(x) = -2x^2 + 4x + 3$
3. $f(x) = -x^2 - 3x + 3$
4. $f(x) = x^2 + 2x - 3$
5. $f(x) = -x^2 + 4x - 1$
6. $f(x) = -2x^2 + 9x + 3$
7. Find a quadratic function such that the vertex is the point $(-3, 4)$ and $(1, 8)$ is another point on the graph.
8. A company finds that its revenue for selling x units of one of its products is given by the quadratic function $R(x) = 200\,x - 0.5\,x^2$, where R is given in dollars. What is the maximum revenue? How many units of this product must be sold so that the maximim revenue is achieved?

Section 3.6 Sum, Difference, Product, Quotient, and Composition of Functions

Let f be a function with domain A. Let g be a function with domain B. Then the

sum, difference, product, quotient and composition of f and g are defined below.
Sum: $(f+g)(x) = f(x) + g(x)$ The domain of $f+g$ is $A \cap B$.
Difference: $(f-g)(x) = f(x) - g(x)$ The domain of $f-g$ is $A \cap B$.
Product: $(fg)(x) = f(x)\,g(x)$ The domain of fg is $A \cap B$.
Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ The domain of $\frac{f}{g}$ is $\{x \in A \cap B | g(x) \neq 0\}$.
Composition: $(fog)(x) = f[g(x)]$ The domain of fog is $\{x \in B | g(x) \in A\}$.
Examples on graphing functions:
1. $f(x) = 2x^2 - 4x + 1$ and $g(x) = 7x$, find $f+g$, $f-g$, $fg, \frac{f}{g}$, and their domains.
2. $f(x) = 3x+1$ and $g(x) = 4x+5$, find $(f+g)(1)$, $(f-g)(2)$,$(fg)(-1)$,$\left(\frac{f}{g}\right)(-2)$, and $(fog)(3)$
3. $f(x) = x^2 - 5$ and $g(x) = x^3 + 1$, find fog, gof, and their domains.
4. Find the domain of the function $H(x) = \frac{\sqrt{x-1}}{x-5}$.
5. For the functions $f(x) = \sqrt{x-2}$ and $g(x) = x+1$, find fog and its domain.
6. For the functions $f(x) = \frac{4}{x+1}$, $g(x) = \sqrt{x^2+1}$, and $h(x) = 3x+2$, find the following: (a) $(f-g)(-2)$, (b)$(gof)(3)$, (c)$(gog)(2)$, and (d) $\left(\frac{g}{h}\right)(-1)$.

Section 3.7 One-to-One Functions and Their Inverses

Let f be a function with domain A. f is said to be **one-to-one** if no two elements in A have the same image.
This means that $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Horizontal Line Test

A function is one-to-one if no horizontal line intersects its graph more than once.
Let f be a one-to-one function with domain A and range B. Then f has an **inverse function**, which is denoted by f^{-1}.
The inverse function has domain B and range A and is given by $f^{-1}(y) = x \equiv f(x) = y$ for each $y \in B$.

Property of Inverse Functions

$f^{-1}(f(x)) = x$ for each x in A and
$f(f^{-1}(x)) = x$ for each x in B.

Examples 1. $f(x) = 2x+3$
2. Is the function $f(x) = x^2 - 2$ one-to-one?
3. Use the Property of Inverse Functions to show that $f(x) = 2x-5$ and $g(x) = \frac{x+5}{2}$ are inverses of each other.
4. Is the function $f(x) = (x-1)^2$ one-to-one?
5. The function $f(x) = 8x+3$ is one-to-one. Find the inverse function f^{-1}.
6. Use the Property of Inverse Functions to determine whether or not the functions $f(x) = 5\,x^3$ and $g(x) = \sqrt[3]{\frac{x}{5}}$
are inverses of each other.

Section 4.1

Polynomial Functions and Basic Graphs

A function of the form
$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_o$, where $a_n \neq 0$ and the exponents

on x are whole numbers, is called a **polynomial function of degree n**.
The number a_n is called the **leading coefficient** and the term $a_n x^n$ is called the **leading term**. The number a_0 is called the **constant term**.
The graph of a polynomial function of the form $P(x) = x^n$ has the same general shape as $y = x^2$ when n is an even number > 2.
The graph of a polynomial function of the form $P(x) = x^n$ has the same general shape as $y = x^3$ when n is an odd whole number greater than 3.
Examples: 1. Sketch the graph of the polynomial $P(x) = (x + 1)^3$ by transforming the graphs of the function $y = x^3$.
2. Sketch the graph of the polynomial $P(x) = x^4 + 1$ by transforming the graphs of the function $y = x^4$.
3. For the polynomial function $P(x) = -11x^9 + 7x^5 - 2x + 3$, identify the leading term, the degree, the leading coefficient, and the constant term.
4. Sketch the graph of the polynomial function $P(x) = x^3 - 4$. Do not plot points, but instead apply transformations to the graph of the polynomial function $g(x) = x^3$.
5. Sketch the graph of the polynomial $P(x) = -x^4 + 8$ by transforming the graphs of the function $y = x^4$.
6. Sketch the graph of the polynomial $P(x) = 2x^3 - 7$ by transforming the graphs of the function $y = x^3$.

A Strategy for Graphing Polynomial Functions

To sketch the graph of a polynomial function $y = P(x)$ of degree three or greater, it is helpful to follow the guideline given below:
1. A **zero** of P is a number c that satisfies $P(c) = 0$. If c is a zero of P, then $x - c$ is a factor pf $P(x)$. Factor the polynomial to find its real zeros. These are the x-intercepts of the graph. If c is a real zero of P and the factor $cx - c$ occurs m times in the factorization of P, then the graph will cross the x-axis at c if m is odd and will not cross the x-axis if m is even. Near the x-intercept c, the graph will have the same general shape as $y = A(x - c)^m$.
2. The y-intercept is given by $P(0)$.
3. Determine the **end behavior** of P. The end behavior is determined by the degree of P and the sign of the leading coefficient.
4. Make a table of values to determine other points on the graph.
5. Plot the points found in steps 1, 2, and 4. Connect the points with a smooth curve and make sure that the curve exhibits the behavior determined in steps 1 and 3. **1.** Use the strategy for graphing polynomial functions to sketch the graph of
$P(x) = x^3 - x^2 - 2x$.
2. describe the end behavior of the polynomial function $P(x) = -4x^6 - x^2 + 11x + 5$.
3. describe the end behavior of the polynomial function $P(x) = x^4 + 2x^3 - x^2 - 2x$.
4. Find the intercepts of the graph of the polynomial function $P(x) = x^3 - 3x^2 + 2x$.
5. Sketch the graph of the polynomial function $P(x) = -x^3 + x^2 + 20x$.

5. Sketch the graph of the polynomial function $P(x) = (x+2)^2(x-2)^2$.

Section 4.2

Polynomial Long Division and Synthetic Division

The **Division Algorithm** states that if $P(x)$ and $D(x)$ are polynomials, with $D(x) \neq 0$, then there are unique polynomials $Q(x)$ and $R(x)$ such that $P(x) = D(x) \cdot Q(x) + R(x)$.

The polynomial $P(x)$ is called the **dividend.** The polynomial $D(x)$ is called the **divisor.** The polynomial $Q(x)$ is called the **quotient**. The polynomial $R(x)$ is called the **remainder.** The remainder $R(x)$ is either 0 or of degree less than the degree of the dividor $D(x)$.

Examples: 1. Find the quotient and remainder for $\dfrac{2x^4 - 8x^2 + 3x - 5}{x-5}$ by using polynomial long division.

2. Synthetic division is a quick method of dividing polynomials, which can be used when the divisor is of the form $D(x) = x - c$. Find the quotient and remainder for $\dfrac{2x^4 - 8x^2 + 3x - 5}{x-5}$ by using synthetic division.

3. Find the quotient and remainder for $\dfrac{3x^3 - 5x^2 + 2x + 1}{x+5}$ by using polynomial synthetic division.

4. Find the quotient and remainder for $\dfrac{x^3 + 3x - 1}{x-4}$ by using polynomial long division.

5. Use polynomial long division to find the uotient and remainder for $\dfrac{6x^3 + 9x^2 + 22x + 8}{3x^2 - 2}$

6. Use synthetic division to find the quotient and remainder for $\dfrac{x^3 + 4x^2 - 3x + 1}{x-2}$

7. Use synthetic division to find the quotient and remainder for $\dfrac{3x^4 + 2x^2 - 3x + 1}{x+3}$

The remainder Theorem: If the polynomial $P(x)$ is divided by $x - c$, then the remainder is $P(c)$.

The Factor Theorem: c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Examples: 1. Use synthetic division and the Remainder Theorem to evaluate $P(-4)$ for $P(x) = 2x^3 - 2x^2 + 11x - 100$. **2.** Use the Factor Theorem to show that $x - 2$ is a factor of the polynomial $P(x) = x^5 - 13x^4 + 57x^3 - 83x^2 - 34x + 120$.

3. Find a polynomial of degree three that has zeros -2, -1, and 3.

4. For $P(x) = 4x^4 - 9x^3 + 8x^2 - 5x + 2$, evaluate $P(2)$ by using synthetic division and the Remainder Theorem.

5. For $P(x) = -5x^4 - 8x^3 + 6x + 3$, evaluate $P(-3)$ by using synthetic division and the Remainder Theorem.

6. Use the Remainder Theorem to determine whether or not $x - 5$ is a factor of $P(x) = 3x^3 - 19x^2 + 21x - 5$.

7. Find a polynomial of degree three that has zeros -1, 4, and 10 by using the Factor Theorem.
8. Let $P(x) = x^3 + 4x^2 - 7x - 10$. Use the fact that $P(2) = 0$ to find the zeros of the polynomial.

Section 4.3
Finding All Zeros of Polynomials

In finding all zeros of a polynomial P, factor the polynomial completely. If a factor $x - c$ appears k times in the complete factorization of $P(x)$, then we say that c is a zero of multiplicity k.
A polynomial of degree $n \geq 1$ has exactly n zeroes, counting multiplicities.
For a polynomial with real cosfficients, complex zeros occur in complex conjugate pairs; that is, if the complex number $a+bi$ is a zero of P, then its complex conjugate $a - bi$ is also a zero of P.
If c is a **zero** of the polynomial P, then $x = c$ is a **root** of the equation $P(x) = 0$.
Examples 1. Factor the polynomial $P(x) = x^3 + 16x$ completely and find all its zeros. State the multiplicity of each zero.
2. Let $P(x) = x^3 + 6x^2 + 2x + 12$. Find all roots of the equation $P(x) = 0$.
3. Find a polynomial of degree 5 with integer coefficients that has zeros $\frac{1}{2}$, $5i$, and $\sqrt{3}i$.
4. Factor the polynomial $P(x) = x^3 - 2x^2 + x$ completely and find all its zeros. State the multiplicity of each zero.
5. Factor the polynomial $P(x) = x^3 + 4x^2 + 9x + 36$ completely and find all its zeros. State the multiplicity of each zero.
6. For $P(x) = x^2 - 4x^2 + 5$ find all zeros, give the multiplicity of each zero, and then write the polynomial in factored form.
7. Find a polynomial of degree three with integer coefficients that has zeros $\frac{2}{3}$ and $8i$.
8. Find a polynomial of degree four with integer coefficients that has zeros 1 and $4 + 3i$, with 1 a zero of multiplicity 2.
9. Factor the polynomial $P(x) = x^4 - 256$ into linear and irreducible quadratic factors with real coefficients and then factor the polynomial completely into linear factors with complex coefficients.

Section 4.4
Holes, Vertical Asymptotes, and Horizontal Asymptotes

A rational function r is a function of the form
$r(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials.
A vertical line $x = a$ is called a **vertical asymptote** of the rational function $y = r(x)$ if
$y \Rightarrow \infty$ or $y \Rightarrow -\infty$ as $x \Rightarrow a^+$ or $x \Rightarrow a^-$.

A horizontal line $y = b$ is called a **horizontal asymptote** of the rational function $y = r(x)$ if
$y \Rightarrow b$ as $x \Rightarrow \infty$ or $x \Rightarrow -\infty$.

Examples: 1. Identify any holes, vertical asymptotes, and horizontal asymptotes for the rational function $r(x) = \dfrac{x+2}{x^2-4}$.

2. Find the vertical and horizontal asymptotes for the function $r(x) = \dfrac{3x+1}{x-3}$.

3. Find the vertical and horizontal asymptotes for the function $r(x) = \dfrac{4x}{x^2-5x+4}$.

4. Find all holes, vertical asymptotes, and horizontal asymp., if any, for the rational function $r(x) = \dfrac{2x}{x+5}$.

5. Find all holes, vertical asymptotes, and horizontal asymp., if any, for the rational function $r(x) = \dfrac{3x^2}{x^2-4}$.

6. Find all holes, vertical asymptotes, and horizontal asymp., if any, for the rational function $r(x) = \dfrac{x-2}{x^2+2x-3}$.

7. Find all holes, vertical asymptotes, and horizontal asymp., if any, for the rational function $r(x) = \dfrac{x^2+1}{x}$.

8. Find all holes, vertical asymptotes, and horizontal asymp., if any, for the rational function $r(x) = \dfrac{5(x-1)}{(x-1)(x^2+4)}$.

Graphing Rational Functions

To sketch the graph of a rational function $r(x) = \dfrac{P(x)}{Q(x)}$ that has vertical and horizontal asymptotes, follow the steps given below:

1. Factor the numerator and denominator if necessary and check for common factors.
2. Find the x- and y-intercepts.
3. Find the vertical and horizontal asymptotes.
4. Determine the behavior near the vertical asymptotes.
5. Find as many additional points as needed to complete the graph.

Examples: 1. Sketch the graph of the rational function $r(x) = \dfrac{x+6}{x-2}$. **2.** $r(x) = \dfrac{3}{x-1}$. **3.** $r(x) = \dfrac{x-3}{x-1}$. **4.** $r(x) = \dfrac{1-2x}{x+1}$. **5.** $r(x) = \dfrac{1}{(x+1)(x-1)}$.