NAME (please print): ________________________________

FINAL EXAM (Sample)

1.(5 pts) Which of these four PDEs is the heat equation, the wave equation, or the Laplace's equation? Write the name of the equation next to it.

$$\begin{array}{llll} (1) & u_t = 2u_{xx} + 5 & (2) & u_{tt} = 3u_{xx} \\ (3) & 2u_t = 3u_{xx} & (4) & u_{xx} + u_{yy} = 0. \end{array}$$

2.(a) (5 pts) How many initial data do we need to prescribe for the wave equation

$$u_{tt} = c^2 u_{xx}, \quad x \in (-\infty, \infty), t > 0,$$

to have a well-defined initial value problem?
(b) (5 pts) Write the general form of the initial data that needs to be prescribed.

3.(10 pts) State the principle of superposition for the homogeneous problem $L(u) = 0$.

4.(10 pts) True of false:
(a) Solutions of the wave equation $u_{tt} = c^2 u_{xx}$ with fixed end points conserve energy.
(b) Solutions of the heat equation do not decay in time.

5.(10 pts) Find the eigenvalues and the corresponding eigenfunctions of the following eigenvalue problem (show your calculation):

$$\begin{aligned} \frac{d^2\phi}{dx^2} &= -\lambda\phi, \quad x \in (0, L) \\ \phi(0) &= 0, \qquad \frac{d\phi}{dx}(L) = 0. \end{aligned}$$

6.(20 pts) Using D'Alambert formula, find the solution of the following initial-value problem for the wave equation (simplify the solution as much as possible):

$$\begin{aligned} u_{tt} &= u_{xx}, \quad x \in (-\infty, \infty),\ t > 0, \\ u(x,0) &= 3, \\ u_t(x,0) &= x. \end{aligned}$$

7.(20 pts) Solve the following initial-boundary-value problem for the heat equation (show your work):

$$\begin{aligned} \frac{\partial u}{\partial t} &= 3\frac{\partial^2 u}{\partial x^2}, \quad x \in (0,1), t > 0, \\ u(0,t) &= 0, \quad u(1,t) = 0, \\ u(x,0) &= 2\sin \pi x + 4 \sin 3\pi x. \end{aligned}$$

8.(20 pts) Solve the Laplace's equation inside a rectangle $0 \le x \le L$, $0 \le y \le H$, with the following boundary conditions (show your work):

$$u(0,y) = \sin\frac{2\pi y}{H}, \quad u(L,y) = 0, \quad u(x,0) = 0, \quad u(x,H) = 0.$$