# Homework 4: Feb 1, 2017

1. (Problem 2.2.1) Show that a linear combination of linear operators is a linear operator.

2. (Problem 2.2.2 (a)) Show that

$$L(u) = \frac{\partial}{\partial x}\left[K_0(x)\frac{\partial u}{\partial x}\right]$$

is a linear operator.

3. (Problem 2.2.3) Show that

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q(u, x, t)$$

is linear if $Q = \alpha(x,t)u + \beta(x,t)$, and, in addition, homogeneous if $\beta(x,t) = 0$.

4. (Problem 2.2.4) IMPORTANT:

<u>Superposition Principle for Nonhomogeneous Problems</u>

(a) Consider $L(u) = f$. If $u_p$ is a particular solution, i.e., $L(u_p) = f$, and if $u_1$ and $u_2$ are two homogeneous solutions, i.e., $L(u_1) = 0$, $L(u_2) = 0$, show that

$$u = u_p + c_1 u_1 + c_2 u_2$$

is another particular solution.

(b) If $L(u) = f_1 + f_2$, where $u_{p_i}$ is a particular solution corresponding to $f_i$, what is a particular solution for $f_1 + f_2$?