NAME (please print): ANSWER KEY IS COMING LATER TONIGHT

Quiz 6 Sample

1. Suppose that $u(x,t) = F(x - ct)$. Evaluate:
(a) $\partial u/\partial t$ at $(x, 0)$,
(b) $\partial u/\partial x$ at $(0, t)$.

2. (a) Suppose that an infinite vibrating string has initial displacement equal to zero, and initial velocity $u_t(x, 0) = x^2$. Find the solution $u(x,t)$.

(b) What is the domain of dependence of a point $(x, 1)$? Sketch it.

3. (a) Solve $u_{tt} = u_{xx}$ for $x \in R$, $t > 0$, with the initial data $u(x, 0) = \sin x$ and $u_t(x, 0) = \cos x$.

(b) Sketch the solution at $t = 1$ s.

(c)Sketch and specify the **Domain of dependence** of point $(x,t) = (2, 3)$ and the **Range of influence** of point $(x_0, 0) = (0, 0)$.

4. (a) Derive the total energy of a vibrating linearly elastic string of length L, modeled by $u_{tt} = c^2 u_{xx}$.

(b) Suppose that the string is fixed at $x = 0$, i.e., $u(0,t) = 0$, and free at $x = L$, i.e., $u_x(L, t) = 0$. What is the rate of change of the total energy of the string? Is the energy conserved?

(c) For the boundary data in (b), find the total energy of the string at any time t if initially the string had zero velocity and its displacement was $u(x, 0) = \sin \frac{\pi x}{L}$?

5. Solve the non-constant coefficient heat equation

$$\frac{\partial u}{\partial t} = 3 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right), \quad r \in (1, 2), t > 0$$

describing heat flow in a circular annulus bounded by the inner circle of radius $r = 1$, and the by the outer circle of radius $r = 2$. The annulus material has thermal diffusivity $k = 3$. Suppose that the initial temperature of the annulus is

$$u(r, 0) = f(r), \;\; r \in (1, 2)$$

and that the inner and outer circular boundary are insulated, i.e.,

$$\frac{\partial u}{\partial r}(1, t) = 0, \quad \frac{\partial u}{\partial r}(2, t) = 0.$$

You may assume that the corresponding eigenfunctions, denoted $\phi_n(r)$, are known and are complete. Write the formula for the solution in terms of the general Fourier series, and specify how would you calculate the coefficients in the generalized Fourier series.