NAME (please print):

# Quiz 7

1. (a) Consider Laplace's equation in $\Omega$. Is it true that the minimum and maximum values of the solution of the Laplace's equation must be attained on the boundary? Prove your statement.

(b) State the Mean Value Property for the Laplace's equation defined on a domain $\Omega$.

2. Consider the Laplace's equations inside a circular disk of radius 1, with the prescribed boundary data equal to:

$$u(1,\theta) = \sin(\theta)\cos(\theta), \quad -\pi \le \theta < \pi.$$

(a) What is the condition on $u$ that you need to impose at $r = 0$ to be able recover the physically reasonable solution?

(b) What are the conditions on $u$ that you need to impose at $\theta = \pi$ and $\theta = -\pi$ to be able recover the physically reasonable solution?

(c ) Solve the above problems for the Laplace's equation using the method of separation of variables by assuming $u(r,\theta) = G(r)\phi(\theta)$. As you are calculating the solution, write explicitly:

(c1) What is the ODE that needs to be solved to find $G(r)$?

(c2) What is the eigenvalue problem (the ODE and data) that needs to be solved for $\phi(\theta)$?

(c3) What is the general solution for $u(r,\theta)$ written in terms of the Fourier series?

(c4) What do you use to calculate the final, particular solution from the general solution?

(c4) Write the explicit form of the final solution of this problem.

Recall: Laplace's equation in polar coordinates is given by:

$$\triangle u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$$

Recall: $\sin(2\alpha) = 2\sin\alpha\cos\alpha$.

Show all your work.