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FLUID-STRUCTURE INTERACTION IN BLOOD FLOW

SUNČICA ČANIĆ

This is a joint work with Andro Mikelić (University of Lyon 1, France), Josip Tambača (University of Zagreb, Croatia), Giovanna Guidoboni (University of Houston, USA), Dr. Craig Hartley (Baylor College of Medicine, USA), Dr. Doreen Rosenstrauch (Texas Heart Institute and The University of Texas Health Science Center at Houston, USA), and Dr. Z. Krajcer (Texas Heart Institute and St. Luke's Episcopal Hospital, USA).

The study of flow of a viscous incompressible fluid through a compliant tube has many applications. One major application is blood flow through human arteries. Understanding wave propagation in arterial walls, local hemodynamics, and temporal wall shear stress gradient is important in understanding the mechanisms leading to various complications in cardiovascular function. Many clinical treatments can be studied in detail only if a reliable model describing the response of arterial walls to the pulsatile blood flow is considered.

It has been well accepted that in medium-to-large arteries, blood can be modeled as a viscous, incompressible Newtonian fluid. Although blood is a suspension of red blood cells, white blood cells, and platelets in plasma, its non-Newtonian nature due to the particular rheology is relevant in small arteries (arterioles) and capillaries where the diameter of the arteries becomes comparable to the size of the cells. In medium-to-large arteries, such as the coronary arteries (medium) and the abdominal aorta (large), the Navier-Stokes equations for an incompressible viscous fluid are considered to be a good model for blood flow.

Devising an accurate model for the mechanical behavior of arterial walls is more complicated. Arterial walls are anisotropic and heterogeneous, composed of layers with different biomechanical characteristics [21], [22], [25], [29]. A variety of different models have been suggested in the literature to model the mechanical behavior of arteries [1], [2], [3], [21], [22], [23], [25], [29], [30]. They range from the detailed description of each of the layers to the average description of the total mechanical response of the vessel wall assuming homogeneous, linearly elastic behavior.

To study the *coupling* between the motion of the vessel wall and pulsatile blood flow, a detailed description of the vessel wall biomechanical properties may lead to a mathematical and numerical problem whose complexity is beyond today's computational capabilities. The nonlinearity of the underlying fluid-structure interaction is so severe that even a simplified description of the vessel wall mechanics assuming homogeneous linearly elastic behavior leads to complicated numerical algorithms with challenging stability and convergence properties. To devise a mathematical model that will lead to a problem which is amenable to numerical methods producing computational solutions in a reasonable time frame, various simplifications need to be introduced. They can be based on the simplifying model *assumptions* capturing only the most important physics of the problem and/or on the simplifications utilizing *special problem features*, such as special geometry, symmetry, and periodicity.

A common set of simplifying assumptions that captures only the most important physics in the description of the mechanical properties of arterial walls includes homogeneity of the material with "small" displacements and "small" deformation gradients leading to the hypothesis of linear elasticity. A common set of special problem features that leads to simplifying models includes "small" vessel wall thickness, allowing a reduction from 3D models to 2D shell models, and cylindrical geometry of a section of an artery where no branching is present, allowing the use of cylindrical shell models. Neglecting bending rigidity of arteries, studied in [18] and [21], reduces the shell model to a membrane model. Further simplifications include axial symmetry of the loading exerted by the blood flow to the vessel walls in the approximately straight cylindrical sections, leading to axially symmetric models with a potential of further reduction to 1D models. One-dimensional models, although a good first approximation to the underlying problem, suffer from several drawbacks: they are not closed (an ad hoc velocity profile needs to be prescribed to obtain a closed system of equations), and the model equations are quasilinear hyperbolic, typically producing shock wave solutions [6], not observed in healthy humans. In particular, the wall shear stress calculated using one-dimensional models is a consequence of the form of the prescribed velocity profile.

Two-dimensional and three-dimensional models of fluid-structure interaction between the incompressible viscous fluid flow and the motion of a linearly elastic cylindrical membrane are rather complex. Often times, additional ad hoc terms of viscoelastic nature are added to the vessel wall model to provide stability and convergence of the underlying numerical algorithm ([28], [29]) or to provide enough regularity in the proof of the existence of a solution ([14], [17], [24], [4]), thereby showing well-posedness of the underlying problem. To this day there is no analytical result proving well-posedness of the fluid-structure interaction problem without assuming that the structure model includes the higher-order derivative terms capturing some kind of viscoelastic behavior ([14], [17], [24], [4]), or with the terms describing bending (flexion) rigidity in elastic shells or plates ([14], [16]). In fact, current literature on well-posedness of the fluid-structure interaction between a viscous incompressible Newtonian fluid and a viscoelastic structure includes many additional simplifying assumptions, such as the smallness of the data [4]; periodic boundary conditions [24], [4]; or flow in a closed cavity [14], [16], [17], not appropriate for the blood-flow application. Thus, the well-posedness of the fluid-structure interaction problem describing blood flow in compliant (elastic or viscoelastic) arteries remains an open problem. However, even in those simplifying problems when the data are infinitesimally small, the higher-order regularizing terms in the structure model play a crucial role in providing the stabilizing mechanism. Thus, ignoring the terms that account for bending rigidity of the vessel walls and/or viscous dissipation might mean oversimplifying the physics, giving rise to a problem which might not have a solution.

Keeping this in mind, we turn to the theory of elastic/viscoelastic shells to model the mechanical properties of arterial walls. We will be assuming that the vessel walls are homogeneous, that the thickness of the wall is small in comparison to the vessel radius, and that the state of stress is approximately plane, allowing us to consider shell theory. The equations of shell theory have been derived by many authors; see [19] and the references therein. Due to variations in approach and rigor, the variety of equations occurring in the literature is overwhelming. Among all the equations of shell theory, the Koiter shell equations appear to be the simplest consistent first approximation in the general theory of thin elastic shells [27], [26]. In addition, they have been mathematically justified using asymptotic methods to be consistent with three-dimensional elasticity [15]. Ciarlet and Lods showed in [15] that the Koiter shell model has the same asymptotic behavior as the three-dimensional membrane model, the bending model, and the generalized membrane model in the respective regimes in which each of them holds. Motivated by these remarkable properties of the Koiter shell model, in [5], [13], we derived the Koiter shell equations for the cylindrical geometry and extended the linearly elastic Koiter model to include the viscous effects observed in the measurements of the mechanical properties of vessel walls [1], [3], [2]. We utilized the Kelvin-Voigt viscoelastic model in which the total stress is linearly proportional to the strain and the time-derivative of strain. More precisely, for a three-dimensional isotropic and homogeneous body, the Kelvin-Voigt model relates the total stress tensor, whose components we denote by t_{kl}, to the infinitesimal strains e_{kl} and the time-derivative of the strains $\partial_t e_{kl}$ through the following relationship [20]:

$$t_{kl} = (\lambda_e + \lambda_v \partial_t) I_e \delta_{kl} + 2(\mu_e + \mu_v \partial_t) e_{kl}, \quad k, l = 1, 2, 3, \tag{1}$$

where λ_e and μ_e are the Lamé constants of elasticity, λ_v and μ_v are their corresponding viscoelastic counterparts, δ_{kl} is the Kronecker delta, and $I_e :=$

$\sum_{i=1}^{3} e_{ii}$. In [5], [13], we show that the fluid-structure interaction algorithm based on the viscoelastic Koiter shell equations, coupled with the Navier-Stokes equations for a viscous incompressible fluid, captures the experimentally measured viscoelastic properties of arterial walls in the human femoral artery and in the canine aorta. Using the a priori estimates based on an energy inequality, coupled with the asymptotic analysis and homogenization theory as used in [11], [9], [10], and [8], we derived an effective, closed fluid-structure interaction model and a fast numerical solver whose solutions capture the viscoelastic properties of major arteries. We show in [5] and [13] that our effective model approximates the original three-dimensional axially symmetric problem to the ϵ^2 accuracy, where ϵ is the aspect ratio of the cylindrical domain (vessel). Our reduced, effective model reveals several interesting features of the coupled fluid-structure interaction problem:

(1) Our model explicitly shows how the leading order viscous fluid dissipation imparts long-term viscoelastic memory effects on the motion of the vessel wall. We show that this does not influence, to the leading order, the viscoelastic hysteresis loop observed in the stress-strain (or the pressure-diameter) measurements of the arterial viscoelastic properties.

(2) Our model shows that the bending rigidity of vessel walls plays a non-negligible role in the asymptotic behavior of the underlying fluid-structure interaction problem. We found that for the parameters describing blood flow through medium-to-large arteries, the leading-order terms in the coupling of the stresses at the vessel wall include not only the membrane terms but also a correction accounting for the bending rigidity of the wall, often times neglected in the description of the mechanical properties of vessel walls.

We developed a fast numerical solver based on the 1D finite element approach and compared the computational solution with the experimental measurements. First, the reduced *elastic* model was tested experimentally using a mock circulatory flow loop with latex tubing, assembled at the Research Laboratory at the Texas Heart Institute. Then, the *viscoelastic* model was compared to the hysteresis measurements of the viscoelastic properties of the human femoral artery and the canine aorta. In both cases, excellent agreement between the experiment and the numerical solution was obtained; see [5].

Our mathematical results have been successfully applied to the study of the performance of vascular prostheses called stents and stent-grafts used in non-surgical treatment of aortic abdominal aneurysm. For a detailed description, please see [7] and [12].

Main results presented in this talk may be found in the publications listed at `www.math.uh.edu/~canic/hemodynamics/publications.html`.

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Department of Mathematics; 622 PGH Building; University of Houston; Houston, Texas 77204-3476 USA

E-mail address: canic@math.uh.edu