QUIZ 11

Math 3321

Name: Solutions **April 17, 2008**
Due Date: April 22, 2008

Question 1. *(2pts) Convert the differential equation*

$$y^{(4)} + xy''' - 5y' + 2x^3y = \sin 2t - e^t$$

into a system of first-order equations.

$x_1 = y$ $\qquad x_1' = x_2$

$x_2 = y'$ $\qquad x_2' = x_3$

$x_3 = y''$ $\qquad x_3' = x_4$

$x_4 = y'''$ $\qquad x_4' = -2x^3x_1 + 5x_2 - xx_4 + \sin 2t - e^t$

$$x' = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2x^3 & 5 & 0 & -x \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sin 2t - e^t \end{pmatrix}$$

Question 2. *(4pts) Find the homogeneous equation with constant coefficients of least order that has the general solution*

$$y = C_1e^{2x} + C_2xe^{2x} + C_3x^2e^{2x} + C_4$$

$e^{2x} \to r = 2$ multiplicity 3 $\qquad (r-2)^3$

$1 \to r = 0$ multiplicity 1 $\qquad r$

Char. eq: $r(r-2)^3 = r(r^3 - 6r^2 + 12r - 8)$

$= r^4 - 6r^3 + 12r^2 - 8r$

Diff. eq: $y^{(4)} - 6y''' + 12y'' - 8y' = 0$

Question 3. *(4pts) Find the solution of the initial value problem:*

$$y''' + y' = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 2.$$

Char. eq: $r^3 + r = 0$

$r(r^2+1) = 0$

$r = 0 \qquad r = \pm i$

General solution:

$$\begin{cases} y = c_1 + c_2 \cos x + c_3 \sin x \\ y' = -c_2 \sin x + c_3 \cos x \\ y'' = -c_2 \cos x - c_3 \sin x \end{cases}$$

Set $x = 0$

$$\begin{cases} c_1 + c_2 = 0 \\ c_3 = 1 \\ -c_2 = 2 \end{cases}$$

$\Rightarrow$ $c_1 = 2$

$c_2 = -2$

$c_3 = 1$

$$\boxed{y = 2 - 2\cos x + \sin x}$$