

# QUIZ 1

Math 3321

Name:

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There are 3 Questions

Question 1. (3pts) True or False?

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(i) The general solution of a second order ordinary differential equation is a three-parameter family of solutions of the given differential equation. two

F

(ii) If a solution of an initial value problem exists, then it is unique.

F

(iii) The ODE

$$xy^3 - y' + 4y'' = e^x$$

is of third order.

second

not necessarily

Question 2. (4pts) Show your work! This question consists of two parts.

(a) Show that each member of the two-parameter family of functions

$$y = c_1 x + c_2 \sqrt{x}$$

is a solution of the differential equation  $2x^2 y'' - xy' + y = 0$ .

$$y = c_1 x + c_2 \sqrt{x}$$

$$y' = c_1 + \frac{c_2}{2\sqrt{x}}$$

$$y'' = 0 - \frac{c_2}{4x^{3/2}}$$

$$\text{LHS} = 2x^2 y'' - xy' + y = 2x^2 \left( -\frac{c_2}{4x^{3/2}} \right) - x \cancel{c_1} - \frac{x c_2}{2\sqrt{x}} + \cancel{c_1 x} + c_2 \sqrt{x}$$

$$= -\frac{c_2 \sqrt{x}}{2} - \frac{c_2 \sqrt{x}}{2} + c_2 \sqrt{x} = 0 = \text{RHS}$$

=> solution

(b) Find a solution of the initial value problem

$$2x^2 y'' - xy' + y = 0$$

$$y(4) = 1, \quad y'(4) = -2,$$

if it exists.  $y = c_1 x + c_2 \sqrt{x}$

$$y(4) = 1 = 4c_1 + 2c_2$$

$$y'(4) = -2 = c_1 + \frac{c_2}{4}$$

$$1 = 4c_1 + 2c_2$$

$$\underline{-8 = 4c_1 + c_2}$$

$$9 = c_2$$

$$c_1 = -2 - \frac{9}{4} = -\frac{17}{4}$$

**Question 3.** (3pts) Find the general solution to the linear differential equation

$$xy' - y = 3x \ln x$$

$$y' - \frac{1}{x} y = 3 \ln x$$

$$\int -\frac{1}{x} dx = -\ln x$$

$$e^{-\ln x} = e^{\ln \frac{1}{x}} = \frac{1}{x}$$

$$\frac{1}{x} y' - \frac{1}{x^2} y = \frac{3}{x} \ln x$$

$$\left(\frac{1}{x} y\right)' = \frac{3}{x} \ln x$$

$$\frac{1}{x} y = \int \frac{3}{x} \ln x dx = 3 \frac{(\ln x)^2}{2} + C$$

$$\boxed{y = \frac{3x}{2} (\ln x)^2 + Cx} \text{ general solution}$$