

QUIZ 3

Math 3321

Name: Solutions

January 31, 2008

Question 1. Find the general solution.

(i) (6 pts) $yy' = xy^2 + x$

(ii) (4 pts) $y' + y = xy^3$

(i) $yy' = xy^2 + x$ ~~is~~ Separable

$$yy' = x(y^2 + 1)$$

$$\frac{y}{y^2 + 1} y' = x$$

$$\int \frac{y dy}{y^2 + 1} = \frac{x^2}{2} + C$$

$$\frac{1}{2} \ln|y^2 + 1| = \frac{1}{2} x^2 + C$$

$$y^2 + 1 = Ae^{x^2}$$

$$\boxed{y^2 = Ae^{x^2} - 1}$$

(ii) $y' + y = xy^3$

$$\frac{y'}{y^3} + \frac{1}{y^2} = x$$

$$-\frac{1}{2}v' + v = x$$

$$v' - 2v = -2x \leftarrow \text{linear}$$

$$(e^{-2x} v)' = -2x e^{-2x}$$

int. by parts

$$e^{-2x} v = -2 \int x e^{-2x} dx = -2 \left[\frac{x e^{-2x}}{-2} + \int \frac{e^{-2x}}{2} dx \right] = x e^{-2x} + \frac{1}{2} e^{-2x} + C$$

$$v = x + \frac{1}{2} + c e^{2x}$$

$$\boxed{y^2 = \frac{1}{x + \frac{1}{2} + c e^{2x}}}$$

Bernoulli

$n=3$

$$v = y^{-3+1} = y^{-2}$$

$$v' = -2 y^{-3} y'$$