QUIZ 8

Math 3321

Name: **March 27, 2008**
Due: April 1

Question 1. *(4pts)*
Solve the system by reducing the augmented matrix to its reduced row echelon form:

$$x_1 + 2x_2 - 3x_3 - 4x_4 = 2$$
$$2x_1 + 4x_2 - 5x_3 - 7x_4 = 7$$
$$-3x_1 - 6x_2 + 11x_3 + 14x_4 = 0$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 2 \\ 2 & 4 & -5 & -7 & 7 \\ -3 & -6 & 11 & 14 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 2 & 6 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & -3 & -4 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 11 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 + 2x_2 - x_4 = 11 \\ x_3 + x_4 = 3 \end{cases} \qquad \begin{cases} x_1 = -2a + b + 11 \\ x_2 = a \\ x_3 = 3 - b \\ x_4 = b \end{cases}$$

Question 2. *(3pts) Solve the homogeneous system*

$$\begin{aligned} 2x_1 - 2x_2 - x_3 + x_4 &= 0 \\ -x_1 + x_2 + x_3 - 2x_4 &= 0 \\ 3x_1 - 3x_2 + x_3 - 6x_4 &= 0 \\ 2x_1 - 2x_2 - 2x_4 &= 0 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 2 & -2 & -1 & 1 & 0 \\ -1 & 1 & 1 & -2 & 0 \\ 3 & -3 & 1 & -6 & 0 \\ 2 & -2 & 2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & -12 & 0 \\ 0 & 0 & 2 & -6 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 = a + b \\ x_2 = a \\ x_3 = 3b \\ x_4 = b \end{cases}$$

Question 3. *(3pts) What condition must be placed on a, b and c so that the system*

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 6y - 11z &= b \\ x - 2y + 7z &= c \end{aligned}$$

has at least one solution.

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 6 & -11 & b \\ 1 & -2 & 7 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & -4 & 10 & c-a \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & -5/2 & \frac{b-2a}{2} \\ 0 & 0 & 0 & c-a+2(b-2a) \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & -5/2 & \frac{b-2a}{2} \\ 0 & 0 & 0 & c+2b-5a \end{array}\right)$$

If $\boxed{0 = c + 2b - 5a}$

then at least one solution