

QUIZ 9

Math 3321

Name:

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Question 1. (.5pt each) Mark each statement "True or False".

F 1. (TRUE/FALSE) The determinant of the square matrix A is the product of the diagonal entries in A .

only if triangular matrix

T 2. (TRUE/FALSE) If two rows of an $n \times n$ matrix A are the same, then $\det(A) = 0$.

F 3. (TRUE/FALSE) If A and B are row equivalent, then $\det(A) = \det(B)$.

Question 2. (3.5pts) Compute the determinant

$$\begin{vmatrix} 6 & 3 & 2 & 0 \\ 0 & -5 & 6 & 1 \\ 3 & 0 & 0 & 0 \\ 4 & -1 & 3 & 0 \end{vmatrix} = 3 \begin{vmatrix} 3 & 2 \\ -5 & 6 \\ -1 & 3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & 2 \\ -1 & 3 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 3 & 2 \\ -1 & 3 \end{vmatrix}$$

$$= -3 (9 + 2)$$

$$= -3 \cdot 11$$

$$= -33$$

Question 3. (5pts) Use the determinant to decide whether the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

has an inverse.

(ii) If the inverse exists, find it.

$$\begin{aligned} (i) \quad \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{vmatrix} &= 1 \begin{vmatrix} -1 & 3 \\ 1 & 8 \end{vmatrix} + 2 \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} \\ &= -8 - 3 + 2(2 + 4) \\ &= -11 + 12 = 1 \Rightarrow \text{invertible} \end{aligned}$$

$$(ii) \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$$