CORRECTION TO "EQUILIBRIUM MEASURES FOR SOME PARTIALLY HYPERBOLIC SYSTEMS"

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In the proof of Lemma 6.6, the sentence beginning with "It follows directly from Lemma 6.3" contains an erroneous deduction: the first and third inequalities in the next line do not follow immediately as claimed. We are grateful to Xue Liu for bringing this issue to our attention.

In spite of this error, Lemma 6.6 remains true as stated; the proof is as follows. (Note that much of this proof is essentially the same as the published proof, but some things must be rearranged.)

Proof of Lemma 6.6. Given $Y \subset M$, write

$$\overline{P}_Y^{\text{span}}(r_2) := \overline{\lim_{n\to\infty}} \frac{1}{n} \log Z_n^{\text{span}}(Y \cap \Lambda, r_2);$$

define $\overline{P}_Y^{\text{sep}}(r_2)$, $\underline{P}_Y^{\text{span}}(r_2)$, and $\underline{P}_Y^{\text{sep}}(r_2)$ similarly. (Since φ is fixed throughout we omit it from the notation.)

Using Lemma 6.3 and the fact that any (n, r_2)-separated subset of $B^u_\Lambda(x, r_1)$ is also an (n, r_2)-separated subset of Λ, we get

$$\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_2) \le \overline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_2) \le \overline{P}^{\text{sep}}_{B^u_\Lambda(x,r_1)}(r_2) \le \overline{P}^{\text{sep}}_{\Lambda}(r_2) \le P(\varphi).$$

Thus to prove Lemma 6.6, it suffices to show that $\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_2) \ge P(\varphi)$. Indeed, it will suffice to show that $\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_2) \ge \underline{P}^{\text{span}}_{\Lambda}(r_3)$ for all $r_3 > 0$.

To this end, fix $r_3 > 0$. By Lemma 6.5, there is $n_0 \in \mathbb{N}$ such that for every $x \in \Lambda$, we have

$$Z_n^{\text{sep}}(B^u_\Lambda(x, r_1), \varphi, r_3/4) \le e^{n_0\|\varphi\|} Z^{\text{sep}}_{n+n_0}(B^u_\Lambda(x, r_1), \varphi, 2r_2).$$

Using this together with Lemma 6.3 gives

$$\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_2) \ge \underline{P}^{\text{sep}}_{B^u_\Lambda(x,r_1)}(2r_2) \ge \underline{P}^{\text{sep}}_{B^u_\Lambda(x,r_1)}(r_3/4) \ge \underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_3/4),$$

and so we can complete the proof of Lemma 6.6 by showing that $\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(r_3/4) \ge \underline{P}^{\text{span}}_{\Lambda}(r_3)$ for all $r_3 > 0$.

For this, we need to use the Lyapunov stability of E^{cs} from Condition (C1) (see also Remark 2.2). Let $\delta > 0$ be given by (C1) with $\epsilon = r_3/4$, and consider for each $y \in \Lambda$ the (relatively) open set $U_y := \bigcup_{z\in B^u_\Lambda(y,r_1)} B^{cs}_\Lambda(z, \delta)$. Since Λ is compact, we have $\Lambda \subset \bigcup_{i=1}^N U_{y_i}$ for some $\{y_1, \dots, y_N\}$. Now for any $x \in \Lambda$, Lemma 6.4 gives $(n - n_1, 3\epsilon)$-spanning sets E_i for $B^u_\Lambda(y_i, r_1)$ such that

$$\sum_{z\in E_i} e^{S_n\varphi(z)} \le Q_2 Z_n^{\text{span}}(B^u_\Lambda(x, r_1), \varphi, \epsilon).$$

We claim that E_i is an $(n - n_1, 4\epsilon)$-spanning set for U_{y_i}. Indeed, for every $z \in U_{y_i}$ we have $[z, y_i] \in B^u_\Lambda(y_i, r_1)$ and hence there is $p \in E_i$ such that $d_n(p, [z, y_i]) < 3\epsilon$. Moreover, $d_n(z, [z, y_i]) < \epsilon$ using Condition (C1) and the fact that $z \in B^{cs}([z, y_i], \delta)$; then the triangle inequality proves the claim. Now writing $E' = \bigcup_{i=1}^N E_i$, we see that E' is an $(n-n_1, 4\epsilon)$-spanning set for Λ, and hence,

$$Z^{\text{span}}_{n-n_1}(\Lambda, \varphi, 4\epsilon) \le Q_2 N Z_n^{\text{span}}(B^u_\Lambda(x, r_1), \varphi, \epsilon).$$

Taking logs, dividing by n, and sending $n \to \infty$ gives $\underline{P}^{\text{span}}_{B^u_\Lambda(x,r_1)}(\epsilon) \ge \underline{P}^{\text{span}}_{\Lambda}(4\epsilon)$, which proves Lemma 6.6. □