Math 4377/6308 Advanced Linear Algebra I Spring 2013

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MIDTERM TEST #2

Thursday, March 28, 2013

You must give complete justification for all answers in order to receive full credit.

Name: ____________________________________

	Points	Possible
Problem 1		/10
Problem 2		/15
Problem 3		/20
Problem 4		/20
Problem 5		/20
Problem 6		/15
Total		/100

1. Let $\{v_1, v_2\}$ be a basis for $\mathbb{R}^2$. Is $\{2v_1 + v_2,\ 2v_1 - v_2\}$ necessarily a basis for $\mathbb{R}^2$? *[10 points]*

2. Determine whether or not each of the following functions $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. If it is not linear, prove that it is not linear. If it is linear, write the matrix $[T]_\beta$, where β is the standard ordered basis for $\mathbb{R}^2$.

(a) $T(a_1, a_2) = (\sin(a_1), a_2)$ *[5 points]*

(b) $T(a_1, a_2) = (a_1, a_2 + 1)$ *[5 points]*

(c) $T(a_1, a_2) = (2a_1 + a_2, 3a_1 - 2a_2)$ *[5 points]*

3. Let V and W be vector spaces and $T\colon V \to W$ a linear transformation. Suppose that T is one-to-one and that S is a subset of V.

(a) State what it means for S to be linearly independent. [*5 points*]

(b) Prove that S is linearly independent if and only if $T(S)$ is linearly independent. [*15 points*]

4. Consider the ordered bases β and β' for $\mathbb{R}^2$ given by

$$\beta = \{(1,1),\ (1,-1)\}, \qquad \beta' = \{(3,1),\ (-1,3)\}.$$

(a) Find the change of coordinate matrix Q that changes β'-coordinates into β-coordinates. *[10 points]*

(b) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation and let $A = [T]_\beta$. Write an expression for $[T]_{\beta'}$. *[10 points]*

5. For each of the following linear transformations, determine whether it is (i) one-to-one, (ii) onto, and/or (iii) an isomorphism.

(a) $T\colon \mathbb{M}_{m\times n}(\mathbb{R}) \to \mathbb{M}_{n\times m}(\mathbb{R})$ given by $T(A) = A^t$. [*10 points*]

(b) $T\colon \mathbb{P}(\mathbb{R}) \to \mathbb{P}(\mathbb{R})$ given by $(T(f))(x) = f'(x)$. [*10 points*]

6. (a) Give a matrix A such that $A \neq 0$ but $A^2 = 0$. [*5 points*]

(b) Give matrices A, B such that $AB = I$ but $BA \neq I$. [*5 points*]

(c) Consider the matrix

$$A = \begin{pmatrix} 5 & 0 & -1 & 2 \\ 2 & 1 & 0 & 3 \\ -1 & 1 & 1 & -2 \\ -3 & 0 & 3 & 4 \end{pmatrix}.$$

Let $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4$ be the standard (column) basis vectors in $\mathbb{R}^4$. Compute $A\mathbf{e}_1$, $A\mathbf{e}_2$, $A\mathbf{e}_3$, and $A\mathbf{e}_4$. [*5 points*]

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