HOMEWORK 1

Due *4pm Wednesday, August 28.*

1. Determine whether each of the following statements is true or false. Justify your answers.
    (a) $\forall a \in \mathbb{Z}$, $\exists n \in \mathbb{Z}$ such that we have $a + n = 10$.
    (b) $\exists n \in \mathbb{Z}$ such that $\forall a \in \mathbb{Z}$ we have $a + n = 10$.
    (c) $\forall a \in \mathbb{R}$, $\exists n \in \mathbb{Z}$ such that we have $a + n = 10$.
    (d) $\forall \mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$, $\exists a, b, c \in \mathbb{R}$ with $(a, b, c) \neq (0, 0, 0)$ such that $a\mathbf{u} + b\mathbf{v} + c\mathbf{w} = \mathbf{0}$. *Hint: the last equation can be turned into a system of two equations in three variables.*

2. Write the logical negation of the sentence in **1.(d)**.

3. Consider the complex numbers $z = 2 - i$ and $w = 1 + 3i$. Write the complex numbers $zw, \bar{w}, \bar{z} + w, |w|$, and $\frac{1}{z}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

4. List all of the elements in the following sets.
    (a) $\{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \leq a < 3,\ b^2 \leq 1\}$
    (b) $\{(c, d) \in \mathbb{Z} \times \mathbb{N} \mid d^3 \leq 8,\ |c| \leq d\}$

5. Draw a sketch of the following subsets of $\mathbb{R}^2$.
    (a) $\{(x, y) \in \mathbb{R}^2 \mid (x + y)(x - 1) = 0\}$
    (b) $\{(x, y) \in \mathbb{R}^2 \mid (y^2 - 4)(x^2 - y^2) = 0\}$

6. For each of the following maps, determine whether it is 1-1, and whether it is onto.
    (a) $T \colon \mathbb{R} \to \mathbb{R}$ given by $T(x) = x^2$
    (b) $T \colon \mathbb{R} \to \mathbb{R}$ given by $T(x) = x^3$
    (c) $T \colon \mathbb{R} \to \mathbb{R}$ given by $T(x) = \begin{cases} x - 1 & x \leq 0 \\ x + 1 & x > 0 \end{cases}$