HOMEWORK 2

Due *4pm Wednesday, September 4.* You will be graded not only on the correctness of your answers but also on the clarity and completeness of your communication. Write in complete sentences.

1. Determine whether or not each of the following is a subspace of $\mathbb{R}^2$. Justify your answer.
   (a) $X_1 = \{(x, y) \in \mathbb{R}^2 \mid x + y = 0\}$
   (b) $X_2 = \{(x, y) \in \mathbb{R}^2 \mid x - 1 = 0\}$
   (c) $X_3 = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\}$
   (d) $X_4 = \{(1, 0), (0, 1)\}$
   (e) $X_5 = \text{span}\{(1, 0), (0, 1)\}$

2. Prove that if X and Y are subspaces of V, then so are $X \cap Y$ and $X + Y$.

3. Show that if $\mathbf{0} \in L \subset V$, then L is linearly dependent.

4. Determine which of the following subsets of $\mathbb{R}^3$ are linearly independent.
   (a) $S_1 = \{(1, 1, 0), (3, 0, 0)\}$
   (b) $S_2 = \text{span}\{(1, 1, 0), (3, 0, 0)\}$
   (c) $S_3 = \{(2, 0, 1), (1, 1, 0), (0, 0, 1)\}$

5. Let V be a vector space, let $S \subset V$ be a spanning set, and let $L \subset V$ be linearly independent.
   (a) Show that if $S \subset S' \subset V$, then S' is spanning.
   (b) Show that if $L' \subset L$, then L' is linearly independent.

6. Prove that the set $\{\sin x, \cos x, \sin(2x)\} \subset C^1(\mathbb{R})$ is linearly independent. *Hint: if $a, b, c \in \mathbb{R}$ are such that $a \sin x + b \cos x + c \sin(2x) = 0$ for every $x \in \mathbb{R}$, then in particular, the equation is true for $x = 0, \pi/4, \pi/2$. Show that this implies that $a = b = c = 0$.*