Math 4377/6308 Advanced Linear Algebra I Fall 2013

Dr. Vaughn Climenhaga, PGH 651A

HOMEWORK 3

Due *4pm Wednesday, September 11.* You will be graded not only on the correctness of your answers but also on the clarity and completeness of your communication. Write in complete sentences.

1. Consider the polynomials $f(x) = 2x^3 - x^2 + x + 3$, $g_1(x) = x^3 + x^2 + x + 1$, $g_2(x) = x^2 + x + 1$, and $g_3(x) = x + 1$. Determine (with proof) whether or not $f \in \operatorname{span}\{g_1, g_2, g_3\}$.

2. Consider the following three vectors in K^3: $u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $u_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and $u_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$. Prove that $\{u_1, u_2, u_3\}$ is a basis for K^3.

3. Let $L \subset V$ be linearly independent. Show that $L \cup \{v\}$ is linearly independent if and only if $v \notin \operatorname{span} L$.

4. Suppose that $Y_1, \dots, Y_m$ are subspaces of V with the property that $V = Y_1 + \cdots + Y_m$. Show that the following are equivalent:

(a) every $v \in V$ can be written in a *unique* way as $v = y_1 + \cdots + y_m$, where $y_i \in Y_i$ for $1 \le i \le m$;

(b) if $y_i \in Y_i$ and $y_1 + \cdots + y_m = \mathbf{0}$, then $y_1 = \cdots = y_m = \mathbf{0}$.

Hint: This is very similar to Proposition 3.6 in the course notes.

5. Let S_1 and S_2 be subsets of a vector space V. Show that $\operatorname{span}(S_1 \cup S_2) = \operatorname{span}(S_1) + \operatorname{span}(S_2)$.

6. Show that if $\{x, y\}$ is a basis for X, then so is $\{x + y, x - y\}$.

7. Let $V = \{f \in \mathbb{P}_3 \mid f(1) = f(2) = 0\}$ be the set of cubic polynomials that vanish at the points 1 and 2.

(a) Show that V is a subspace of $\mathbb{P}_3$.

(b) Determine $\dim V$ by finding a basis for V.