Math 4377/6308 Advanced Linear Algebra I Fall 2013

Dr. Vaughn Climenhaga, PGH 651A

HOMEWORK 6

Due *4pm Wednesday, October 9.* You will be graded not only on the correctness of your answers but also on the clarity and completeness of your communication. Write in complete sentences.

1. Construct two 2×2 matrices A and B such that $AB = 0$ but $BA \neq 0$.

2. Let $D \in \mathbb{M}_{n\times n}(K)$ be a *diagonal* matrix – that is, $D_{ij} = 0$ whenever $i \neq j$. Write $d_i = D_{ii}$ for the ith diagonal entry of D, and let $A \in \mathbb{M}_{n\times n}(K)$. Show that DA is obtained from A by muliplying the ith row of A by d_i, and AD is obtained from A by multiplying the jth column of A by d_j.

3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x, y) = (3x - y, 2x + 6y)$.
(a) Write the matrix of T relative to the standard basis on $\mathbb{R}^2$.
(b) Let $v_1 = \left(\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right)$ and $v_2 = \left(\begin{smallmatrix} 1 \\ -2 \end{smallmatrix}\right)$, and write the matrix of T relative to the basis $\{v_1, v_2\}$.

4. Consider the bases $\{f_1, f_2, f_3, f_4\}$ and $\{g_1, g_2, g_3\}$ for $\mathbb{P}_3$ and $\mathbb{P}_2$, respectively, where

$$\begin{aligned} f_1(x) &= 2x, & g_1(x) &= 1 - x, \\ f_2(x) &= 1 - x^2, & g_2(x) &= 1 + x^2, \\ f_3(x) &= 2x + x^2, & g_3(x) &= x - x^2, \\ f_4(x) &= x^2 + 2x^3. \end{aligned}$$

Let $T\colon \mathbb{P}_3 \to \mathbb{P}_2$ be the linear map given by differentiation. For $1 \leq j \leq 4$, write Tf_j as a linear combination of g_1, g_2, g_3, and use this to write the matrix of T relative to the bases given above.

5. Let V, W, X be vector spaces and let $S \in \mathbb{L}(V, W)$, $T \in \mathbb{L}(W, X)$, so the composition TS is in $\mathbb{L}(V, X)$.
(a) Prove that if TS is 1-1, then S is 1-1. Must T be 1-1?
(b) Prove that if TS is onto, then T is onto. Must S be onto?
(c) Prove that if T and S are isomorphisms, then TS is an isomorphism.