Math 4377/6308 Advanced Linear Algebra I Fall 2013

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HOMEWORK 8

Due *4pm Wednesday, October 23.* You will be graded not only on the correctness of your answers but also on the clarity and completeness of your communication. Write in complete sentences.

1. Let $T \in \mathbb{L}(V)$ be nilpotent and let $P \in \mathbb{L}(V)$ be a projection.
(a) Show that 0 is an eigenvalue of T, and that it is the only eigenvalue.
(b) Show that the only possible eigenvalues of P are 0 and 1. Show that if P is not the identity or the zero transformation, then 0 and 1 are both eigenvalues.

2. Consider the matrix $A = \begin{pmatrix} -4 & -6 & 3 \\ 2 & 4 & -2 \\ -2 & -2 & 1 \end{pmatrix}$
(a) Show that $\lambda_1 = 0$, $\lambda_2 = -1$, and $\lambda_3 = 2$ are eigenvalues of A by finding eigenvectors $v_1, v_2, v_3 \in \mathbb{R}^3$ such that $Av_j = \lambda_j v_j$.
(b) Let $Q = [v_1 \mid v_2 \mid v_3] \in \mathbb{M}_{3\times 3}$ be the matrix whose column vectors are the eigenvectors v_j. Compute $Q^{-1}AQ$, and verify that this is a diagonal matrix whose diagonal entries are the eigenvalues of A.

3. Let x_n be the sequence of integers defined by $x_0 = x_1 = x_2 = 1$ and the recursive relationship

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3}.$$

Write down a 3×3 matrix A whose powers can be used to compute x_n, and explain how x_n can be obtained from A^n. *Hint: Let $v_n \in \mathbb{R}^3$ be the vector with components x_n, x_{n-1}, x_{n-2}, and find a relationship between v_n and v_{n-1}.*

4. Use the formula for determinant of a 2×2 matrix to prove that $\det(AB) = \det(A)\det(B)$ for any $A, B \in \mathbb{M}_{2\times 2}$.

5. Given each of the following sets of four points in $\mathbb{R}^2$, compute the area of the parallelogram with vertices at those four points.
(a) $(0,0)$, $(2,1)$, $(1,1)$, and $(3,2)$
(b) $(0,0)$, $(3,2)$, $(2,4)$, and $(-1,2)$
(c) $(1,1)$, $(2,2)$, $(0,3)$, and $(-1,2)$