Math 4377/6308 Advanced Linear Algebra I Fall 2013

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HOMEWORK 9

Due *4pm Wednesday, November 13.* You will be graded not only on the correctness of your answers but also on the clarity and completeness of your communication. Write in complete sentences.

1. (a) Let $D = \operatorname{diag}(\lambda_1, \dots, \lambda_n)$ be diagonal. Show that $\det D = \prod_{i=1}^n \lambda_i$.

(b) Let $A \in \mathbb{M}_{n\times n}$ be upper triangular, and show that $\det A = \prod_{i=1}^n A_{ii}$.

2. Use row reduction to compute the determinant of $A = \begin{pmatrix} 0 & 4 & -1 & 1 \\ -3 & 1 & 1 & 2 \\ 1 & 0 & -2 & 3 \\ 2 & 3 & 0 & 1 \end{pmatrix}$.

3. Determine the signs of the following permutations on 6 symbols.
(a) $\pi = (2, 1, 3, 4, 5, 6)$
(b) $\pi = (4, 5, 2, 1, 6, 3)$
(c) $\pi = (3, 5, 2, 4, 6, 1)$

4. Let $A \in \mathbb{M}_{n\times n}$.
(a) Show that $\det(\lambda A) = \lambda^n \det A$ for any $\lambda \in K$.
(b) Say that A is *skew-symmetric* if $A^t = -A$. Suppose that A is skew-symmetric and invertible. Show that n is even.

5. Prove that $\lambda \in K$ is an eigenvalue of A if and only if $\det(A - \lambda I) = 0$.