Math 4377/6308 Advanced Linear Algebra I Fall 2013
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Test 2 Review

You are responsible for all the material covered through the end of Lecture 16 on Wednesday, October 23. The test will focus primarily on the material covered since Lecture 10 on Wednesday, October 2.

As with the first test, you are responsible for knowing the definitions of all the terms below. Also listed are some common procedures you should know how to do and some general results you should know. The test will be similar in structure to the first test, and will include definitions, short proofs, and simple calculations.

Representing linear maps with matrices.

- Finding matrix of linear map relative to standard bases for K^n and K^m (Theorem 10.1)
- Finding matrix of linear map relative to any bases β and γ

Change of coordinates.

- Interpreting commutative diagrams
- Isomorphism $I_\beta \colon K^n \to V$ corresponding to a basis β for V
- Finding change-of-coordinates matrix I_β^γ that transforms β-coordinates to γ-coordinates
- Finding $[v]_\gamma$ using I_β^γ and $[v]_\beta$
- Finding $[T]_\gamma$ using I_β^γ and $[T]_\beta$: conjugate (similar) matrices

Nilpotent operators, projections.

- Definition of nilpotent ($T^k = \mathbf{0}$)
- Two equivalent definitions of projection: one in terms of R_T, N_T, the other in terms of $T^2 = T$.
- Strictly upper triangular matrices are nilpotent

Dual spaces.

- Interpreting dual spaces as row vectors (when elements of V are column vectors)

Row and column rank.

- Definition of row space, column space, row rank, column rank.
- Row rank and column rank are equal.

Eigenvectors and eigenvalues.

- Definition of eigenvector and eigenvalue
- Eigenspace for λ is nullspace for $A - \lambda I$
- Finding eigenvectors for a given eigenvalue

- Example of a linear transformation in $\mathbb{R}^2$ with no eigenvalues
- Every linear transformation in $\mathbb{C}^n$ has an eigenvalue (Proposition 14.4)

Determinants

- Deriving formula for 2×2 determinant by trying to row reduce to the identity
- Interpretation of 2×2 determinant as signed area of parallelogram
- Row reducing a 3×3 matrix, derivation of formula for determinant
- Interpretation of 3×3 determinant in three different ways: Laplace (cofactor) expansion, sum over permutations, signed volume of parallelepiped
- Sign of permutations, orientation of $v_1, \ldots, v_n \in \mathbb{R}^n$