ERRATA: *Introduction to Probability with Mathematica*, Edward P. C. Kao
World Scientific (https://www.math.uh.edu/~edkao >publications>ERRATA)
or, at any web browser, just type: "edward p. kao, homepage" 9/14/2022

P. xi, L. -4: "vintage" ⟵ "vantage" "an user" ⟵ "a user"

P. 6, L. -10: "$\binom{6}{5}$" ⟵ "$\binom{6}{4}$" (I thank Mr. Travis Schlicht for pointing out this error)

P. 6, L. -8: "$\cdots \binom{6}{5} = 18$" ⟵ "$\cdots \binom{6}{4} = 36$" (I thank Mr. Travis Schlicht for pointing out this error)

P. 7, L. 13: "15^6" ⟵ "6^{15}" (I thank Mr. Travis Schlicht for pointing out this error)

P. 7, L. 15: "15^3" ⟵ "3^{15}"

P. 7, L. 16: "$15^6 - 15^3$" ⟵ "$6^{15} - 3^{15}$"

P. 11, L. 13: "... unit. Let" ⟵ "unit, we let"

P. 11, L. -3: "x_1" ⟵ "x_i"

P. 23, L. 12: "$\bigcap_{i=1}^{n} E_i,$" ⟵ "$\bigcap_{i=1}^{n} E_i^c,$"

P. 27, L. - 0: Add the following at the end of Example 7: (Consider only the frist two cards being dealt to you and the dealer.)

P. 38, Last time of the Table: "(2,3)" ⟵ "(2,4)"

P. 39, L. 6 : "0.0131" ⟵ "0.0574"

P. 45, L. 3: the part "$t \to 1$" can be removed.

P. 45, Illustration 8: The Mathematica output sholud be replaced by the following:

```
chapter2_illustration8.nb

nE = Coefficient[Expand[(t x1 + x2 + x3)^4 (x1 + t x2 + x3)^5 (x1 + x2 + t x3)^6], {t^8}];

nE1 = nE /. {x1 → 1, x2 → 1, x3 → 1}
{823680}

pE = nE1/3^15 // N
{0.0574037}
```

P. 53, L. - 8: "$P(AB)$" $\longleftarrow$ "$P(BC)$"

P. 56, L. - 3: "$P(F)$" $\longleftarrow$ "$P(G)$"

P. 60, L. 8: Remove "(a)"

P. 63, L. 13: Before (a), add "Let A denote that a person is accident prone"

P. 63, L. - 6: "From part (a)" $\longleftarrow$ "Using the result obtained from (a)"

P. 70, L. - 10: "Illustration 3" $\longleftarrow$ "Illustration 1"

P. 74, L. - 7: "$n(\Omega_n)$" $\longleftarrow$ "$n(\Omega_x)$"

P. 75, L. - 3: "vintage" $\longleftarrow$ "vantage"

P. 79, L. 4: "show" $\longleftarrow$ "shown"

P. 80, L. 7: "Example 11" $\longleftarrow$ "Example 14"

P. 80, L. - 7: "Example 12" $\longleftarrow$ "the example"

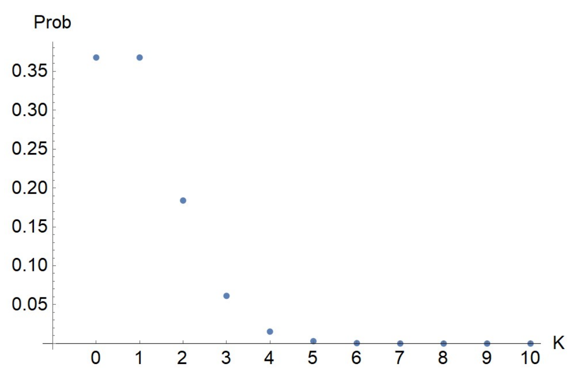
PP, 79 - 82: Two illustrations (along with their contents) should be resequenced:

Illustration 1 (incorrectly stated as Illustration 3)
Illustration 3 (incorrectly stated as Illustration 1)

P. 72, L. 1: Remove "$P_0 \equiv P_0$"

P. 81, Illustration 2: The Mathematica output shown in the text is incorrect. The following is the correcton output.

```
BN = 10;

PN = Accumulate[Table[(-1)^n (1/n!), {n, 2, BN}]];

x = Rest[FoldList[Times, 1, Table[1/(BN - (k - 1)), {k, 1, BN}]]];

y = Table[Binomial[BN, k], {k, 1, BN}];

h = Reverse[Drop[Join[{1, 0}, PN], -1]];

PK = Join[{PN[[BN - 1]]}, h*x*y];

Total[PK]

1

ListPlot[PK, Ticks → {Table[{i, i - 1}, {i, BN + 1}], Automatic},
 AxesLabel → {"K", "Prob"}, AxesStyle → {Black, Medium},
 LabelStyle → {Black, Medium}]
```



P. 118, LL. 5 - 8: "X_n" ⟵ "X_i"

P. 118, L. 7: "nth" ⟵ "ith"

P. 135, L. 5: "An Useful Ways" ⟵ "A Useful Way"

P. 177, L. -13: "a car run" ⟵ "a car will run"

P. 177, L. -11: "5000-miles" ⟵ "5000-mile"

P. 177, L. -8: "it the person" ⟵ "if the person"

P. 202, L. 9: Remove "×(change of variable with $y = x/z$)"

P. 205, L. 10: "Example 15" ⟵ "Example 16"

P. 211, L. 12: " in" ⟵ "is"

P. 229, L. -1: "$(x, ydx$" ⟵ $(x, y)dx$"

P. 230, L. -10: "b, and" $\longleftarrow$ "b, c, and"

P. 233, L. 1: "If" $\longleftarrow$ "Suppose X and Y are two random variables. If"

P. 241, L. -3: "2π" $\longleftarrow$ "$\sqrt{2\pi}$"

P. 246, L. 4: "arrival" $\longleftarrow$ "arrivals"

P. 246, L. -5: "$N(Y)$" $\longleftarrow$ "$N(Y)|Y$"

P. 254: L. -4: The following (known as 8.6 Kolmogorov's Poissonization) should be inserted into the book after Section 8.5. Section 8.6 shown in the book should be renamed as Section 8.7.

8.6 Kolmogorov's Poissonization

Combining many ideas presented in the earlier chapters/sections about Poisson random variables, we are ready to elaborate a powerful yet simple approach due to A. N. Kolmogorov to tackle difficult combinatorial problems. In section 6.4, we have shown that if the interval arrival time of events follows an exponential distribution, then $N(t)$ is the counting random variable representing the number of arrivals of events in the interval $(0, t]$. As t is a time index, $N(t)$ is commonly known as the Poisson process. In Section 5.4, we touch upon the *splitting* property of the Poisson. To elaborate on this a bit more, we assume that for each occurrence of a Poisson event, there is a probability p_i that the event of type i, where $p_1 + \cdots + p_n = 1$. Let $N_i(t)$ be number of type i arrivals over $(0, t]$. The splitting property implies that $N_i(t) \sim \mathit{pois}(p_i\lambda)$, $i = 1, ..., n$, where λ is the parameter of $N(t)$. Moreover, $\{N_i(t)\}_{i=1}^n$ are mutually independent.

Assume that random variable $X_i \sim \mathit{expo}(1)$. Then

$$T = X_1 + \cdots + X_M$$

where X_i is the interarrival time of the Poisson process M. We note each X_i may be of event type i with probability p_i. Applying Wald's equation (Problem 38 of Chapter 8), we have

$$E(T) = E(X_i)E(M) = 1 \times E(M) = E(M)$$

Let T_i denote the first time the process M generates a event of type i. Then

$$T = \max_{j=1,\ldots,n} T_j$$

Then

$$\{T \leq t) = \{T_i \leq t, \quad \forall i = 1, ..., n\}$$

where

$$P(T_i \le t) = 1 - e^{-p_i t} \qquad i = 1, ..., n$$

It follows by (4.2) that

$$E(M) = \int_0^\infty P(T > t) = \int_0^\infty \left(1 - \prod_{i=1}^n P(T_i \le t)\right) dt$$

By Problem 31 of Chapter 4, we also find

$$E(M^2) = \int_0^\infty 2tP(T > t) \qquad (1)$$

Hence we can find $Var(M)$.

Example 22. In Example 14 of Chapter 4, we have the coupon collection problem. In Illustration 3, we specifically look at the numerical example involving 52 distinct types of coupon with equal probability of acquiring each one type, i.e., $p_i = \frac{1}{52}$ for $i = 1, ..., 52$. There we use the inclusion-exclusion formula to compute the PMF for the M.

Now we use the Poissonization approach instead and find

$$E(M) = \int_0^\infty \left(1 - (1 - e^{-\frac{1}{52}t})^{52}\right) dt$$

In Illustration 10, we find $E(M) = 235.978$. We compare this result with that obtained in illustration 3 of Chapter 4. The results are identical to the third decimal place. Using (1), we also find that $Var(M) = 4396.4$. □

Example 23. Consider a variant of the coupon collection problem where there are distinct coupon types, called type 2,...,12, where $p_j = \frac{j-1}{36}$, $j = 2, ..., 7$ and $p_j = p_{14-j}$, $j = 8, ..., 12$. What is the expected number of coupons M needed to collect a complete set?

Using the Poissonization approach, we compute

$$E(M) = \int_0^\infty \left(1 - \prod_{i=2}^{12} (1 - e^{-p_j t})\right) dt$$

In Illustration 11, we see that the MATHEMATICA output yields $E(M) = 61.2174$.□

Consider another class of problem involving the set of events $\{E_i)_{i=1}^n$. Event i will occur with probability p_i. Let i^* be a specific event from the set $\{1, ..., n\}$. Let event G be the event that all (other) events $\{E_j\}$ will occur before the occurrence of event i^*. How to find $P(G)$?

Using the Poissonization idea, we conclude that the density at event G will occur at $(t, t+dt)$ is

$$f_G(t)dt = \left(\prod_{\substack{i=1,\ldots m \\ i \neq i*}} (1 - e^{-p_i t}) \right) \times (p_{i^*} e^{-p_{i^*} t})dt$$

Thus

$$P(G) = \int_0^\infty f_G(t)dt.$$

Example 24. Consider a variant of a crap game involving the rolling of two dice repeatedly. Let G be the event that a total of 2, 3, 4, 5, 6, 8, 9,, 10, 11, and 12 will be rolled before rolling a 7?

Here, we have $i^* = 7$ and hence

$$f_G(t)dt = \left(\prod_{\substack{i=2,\ldots 12 \\ i \neq 7}} (1 - e^{-p_i t}) \right) \times (p_7 e^{-p_7 t})dt$$

The Illustration 12, we find $P(G) = 0.0052577$. □

P. 261, L1 : "Example 20" $\longleftarrow$ "Example 21"

P. 261, before **Problems:** Add the following illustrations:

Illustration 10.

```
EM = Integrate[(1 - (1 - Exp[-(1/52) t])^52), {t, 0, ∞}] // N
```

235.978

```
EM2 = Integrate[2 t (1 - (1 - Exp[-(1/52) t])^52), {t, 0, ∞}] // N
```

60082.1

```
VarM = EM2 - EM^2
```

4396.4

From Illustration 3, Chapter 4 of the text:

```
BN = 52;

fn[i_, n_] = Binomial[BN - i, i] * ((BN - i) / BN)^n * (-1)^(i+1);

pn[n_] := Total[Table[fn[i, n - 1], {i, 1, (BN - 1)}]] - Total[Table[fn[i, n], {i, 1, (BN - 1)}]];

EMa = Total[Table[n * pn[n], {n, 52, 1000}]] // N
```

235.978

□

Illustration 11. The following is related to Example 21 involving the coupon collection problem with unequal probability.

```
p[j_] := (j - 1)/36 /; j ≤ 7

p[j_] := p[14 - j] /; 8 ≤ j ≤ 12

Table[p[j], {j, 2, 12}]
```

$\left\{\frac{1}{36}, \frac{1}{18}, \frac{1}{12}, \frac{1}{9}, \frac{5}{36}, \frac{1}{6}, \frac{5}{36}, \frac{1}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36}\right\}$

```
g[t_] := 1 - ((1 - Exp[-p[2] t]) (1 - Exp[-p[3] t]) (1 - Exp[-p[4] t]) (1 - Exp[-p[5] t]) ×
    (1 - Exp[-p[6] t]) (1 - Exp[-p[7] t]) (1 - Exp[-p[8] t]) (1 - Exp[-p[9] t]) (1 - Exp[-p[10] t]
    (1 - Exp[-p[11] t]) (1 - Exp[-p[12] t]));

EM = Integrate[g[t], {t, 0, ∞}] // N
```

61.2174

□

Illustration 12. A variant of the crap game described in Example 22.

```
p[j_] := (j - 1)/36 /; j ≤ 7

p[j_] := p[14 - j] /; 8 ≤ j ≤ 12

Table[p[j], {j, 2, 12}]

{1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36}

f[t_] := (1 - Exp[-p[2] t]) (1 - Exp[-p[3] t]) (1 - Exp[-p[4] t])
   (1 - Exp[-p[5] t]) (1 - Exp[-p[6] t]) (1 - Exp[-p[8] t])
   (1 - Exp[-p[9] t]) (1 - Exp[-p[10] t]) (1 - Exp[-p[11] t])
   (1 - Exp[-p[12] t]) (p[7] Exp[-p[7] t])

∫_0^∞ f[t] dt // N

0.0052577
```

□

P. 266, Problem 39: It should be labelled as Problem 40.

P. 266, Problem 40: It should be labelled as Problem 39. "$\frac{m!}{m}|$." ⟵ "$\frac{m!}{m^m}$."

P. 319, L. 2: "normal" ⟵ "normally"

P. 267, L. 8: Add the following at the end of paragragh:

The Poissonization approach for solving a class of combinatorial problems is inspired by Tijms [9, p. 290-293]. Examples 22-23 are based on those given in his text.

P. 267, L. -0: Add the following references

9. Tijms. H. *Probability: A Lively Introduction*, Cambridge University Press, 2018.