UH - Math 4377/6308 - Dr. Heier - Fall 2010
HW 1
Due 09/01, at the beginning of class.

Use regular sheets of paper, stapled together.
Don't forget to write your name on page 1.

1. (1 point) Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 6\}$. Explicitly write down the sets

$$A \cup B, \ A \cap (B \cup C), \ B \cap (A \backslash B), \ A \times C.$$

2. (2 points) Let $x, y \in \mathbb{Z}$. Prove or disprove that the following relations are equivalence relations.

(a) $x \sim y$ if and only if $x - y$ is negative.

(b) $x \sim y$ if and only if $x - y$ is even.

3. (3 points) Let $f : \{0, 1, 2, 3, 4\} \to \mathbb{N}, n \mapsto n^2 - n$.

(a) Find the domain, codomain and range of f.

(b) Is f one-to-one?

(c) Is f onto?

4. (1 point) Let a be an arbitrary element in a field. Prove that the additive inverse $-a$ is unique. (Hint: You may use without proof the Cancellation Laws Theorem.)

5. (1 point) Let $z = 1 + 4i$, $w = 1 - i$. Write $\bar{z}$, $z + w$, zw, $|z|$, $\frac{1}{z}$ in the form $a + bi$.

6. (1 point) Solve $z^2 - 4z + 20 = 0$ in $\mathbb{C}$.

7. (1 point) Describe the plane in $\mathbb{R}^3$ through $(1, 2, 3)$, $(2, 0, 1)$, $(0, 1, 0)$ as discussed in class.

8. (1 extra credit point) Let $x, y \in \mathbb{Z}$. Let $x \sim y$ if and only if $y + 4x$ is an integer multiple of 5. Prove that $\sim$ is an equivalence relation.